## ENGR121 Test One

During lecture, 4 April 2019

## Surname:

First Name:

## Student Number:

Please use the spaces provided in this test booklet next to the questions, to give your answers. You may use page five for rough working or for answers if you need more space, plus the reverse sides of all pages.

Attempt all SEVEN questions. The first six questions are of equal value, seven marks each. The last question is worth eight marks. The marks for parts of questions are given in square brackets, e.g. [1].
Silent calculators may be used. A table of formulae is provided.

Page totals, for marking use only

| Page | Mark | max |
| :--- | :--- | :--- |
| p.2 |  | 21 |
| p.3 |  | 14 |
| p.4 |  | 15 |
| Total |  | 50 |

1. State whether each of the following is true or false [1 mark each]:
(a) $-11 \in \mathbb{Z}$
(b) $\pi \in \mathbb{Q}$
(c) $11.1 \in \mathbb{N}$
(d) $11.11 \in \mathbb{Q}$
(e) $\sqrt{2} \in \mathbb{P}$
(f) $\mathbb{Z} \subset \mathbb{R}$
(g) $\mathbb{Q} \cap \mathbb{R}=\mathbb{Q}$
2. Simplify where possible the following operations on sets:
(a) $A \cap \phi$
[1]
(b) $B \cup \bar{B}$
[1]
(c) $A \cap A$
(d) $\overline{\mathbb{E}}$
[2]
(e) $B \cup(B \cap A)$
[2]
3. (a) Sketch a graph of the function
$f(x)=2 x-2$.
(b) Is the function $f(x)=x^{2}+1$ one-toone?
(c) Is the function $f(x)=2 x-\pi$ one-to-one?
$\qquad$
(d) Can a function be one-to-many?
(e) If $f(x)=1+1 / x$, find the inverse function $f^{-1}(x)$.
(f) Write down the composition $f(g(x))$ if $f(x)=\sin x$ and $g(x)=1+x^{2}$.
4. (a) Write down the graphical symbol for a NOT gate.
(b) Write down the truth table for an OR gate with inputs A and B .
[1]
(c) Construct the truth table for $A \cdot B+\bar{A}$.
5. (a) Write the disjunctive normal form for a boolean expression that has the truth table

| $A$ | $B$ | $C$ | $X$ |
| :---: | :---: | :---: | :---: |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 0 |

(b) Use the Fundamental Laws of Boolean algebra, together with $B+1=1$, to prove that

$$
A+A \cdot B=A
$$

(d) Draw a circuit diagram for $A+B \cdot C$ using AND and OR gates.
(e) Simplify the logical expression

$$
A \cdot B+\bar{A} \cdot B
$$

6. (a) Simplify $2 a^{3} / a^{2}$.
[1] 7. (a) Solve the polynomial equation $x^{3}=x^{2}+x$.
(b) Find the roots of $x+1=3 x-3$, showing your working, without using a calculator.
(c) Solve the quadratic equation
$x^{2}+x-2=0$ using any method except a calculator. Show your working.
[2]
(b) Express the following, using only conjunction and negation:

$$
A \cdot B+\bar{A} \cdot \bar{B}
$$

(De Morgan's Laws and complement laws will help here)
(d) Solve the inequality $x^{2}<1$

Use this page and the other side for rough working if needed.

