

Dependencies in Insurance Modeling: An Overview

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Abstract

This literature review summarizes the results from a collection of research papers that relate to modeling insurance claims and the processes associated with them. We consider work by more than 55 authors, published or presented between 1971 and 2008.

Key words: claim sizes; number of claims; multivariate dependency;

1 Motivation

Consider an insurance company where customers may have several insurance policies of different types (for example: fire, health, vehicle, etc). It is unlikely that claims to different policies are independent. A policy holder involved in a car accident will be likely to make claims on both their vehicle and health policies; a fire may spread from one property to another resulting in claims from two or more policy holders; a single event may result in an ongoing series of claims against a single policy.

Given a portfolio of policies it is desirable to accurately forecast the expected liability of those policies. To assume that different policies are independent makes computation of these liabilities straightforward but reduces the accuracy of the estimation. It is therefore worthwhile to develop tractable models with dependence between claims in order to improve the accuracy of the estimation of the costs of servicing different policies.

This literature review was undertaken to establish an understanding of the common approaches to modeling insurance claims and the processes associated with them. We wish to build upon this understanding to later construct a model that incorporates dependence between claim sizes and the frequency of claims. We propose a possible taxonomy of the research on insurance claims, aiming to classify the approaches used in the modeling.

This review considers 37 journal articles, 2 presentations and 4 other papers. Four of the articles were published in the 1970's, the rest of the work was published or presented between 1997 and 2008. Approximately half the articles were published between 2000 and 2005. This review covers writings by more than 55 authors some of whom have made a significant contribution to this field of study.

1.1 Definitions and Common Terms

The study of cost generating processes has been approached from numerous perspectives and contexts. Each context brings its own terminology to the problem. We shall endeavor to use standardized terms in this review.

We shall refer to *claims* as the triggers that accrue costs against the insurer. These may be referred to in the literature as events, business, shocks, damage, claims, policies or risks.

The *cost* of a claim is the magnitude of the effect associated with it. These may be referred to in the literature as magnitudes, loss, size or amount of damage.

A *policy* grants the policy holder the right to make claims and obliges the policy issuer to accept claims.

We shall refer to the *type* of a claim as the group that it is classified as belonging to. Unless stated otherwise types are non-overlapping and therefore partition the set of all claims. These may be referred to in the literature as groups, portfolios, class, book or businesses.

Many approaches use processes to describe how claims occur. It is necessary to distinguish between two types of processes, *occurrence processes* and *claim processes*. Occurrence processes are those that result in an event. Depending on the model this event may correspond to a claim, to claims or to a non-claim event. Claim processes are a special type of occurrence process: those that always result in a claim of the specified type.

In the context of life insurance the *risk* of a policy is the likelihood that the policy will generate a claim in the time period that the model is considering. An increase in the *risk to the insurer* implies increased uncertainty for the insurer as to the total cost of claims in the time period that the model is considering.

It is convenient to establish certain definitions beforehand.

A *copula* is a multivariate distribution with simple marginal distribution for all the random variables (frequently uniform on the interval $[0, 1]$). Copulas are used because it is often easier to model dependency between tractable random variables and then to transform to the distributions of interest, rather than to start with an arbitrary multivariate distribution.

The *probability of ruin* is of particular interest in the actuarial literature. Ruin occurs when the surplus (defined as starting capital plus policy premiums less the cost of claims) falls below zero. The accrual of policy premiums are considered to

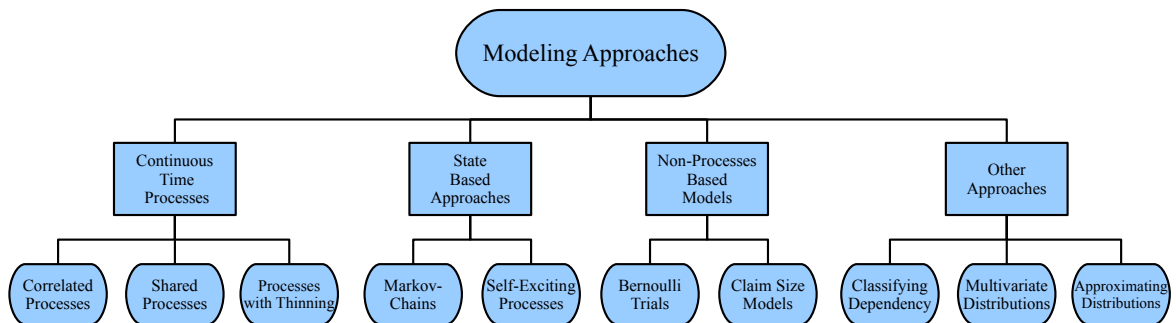
be deterministic. The cost of claims will have some distribution function which is dependent on the distribution of the number of claims and the distribution of the cost of claims.

Some actuarial papers use *stop-loss premium* as a measure of risk. A stop-loss premium is paid by an insurer to a reinsurer to limit or control their total insurance liability. Higher stop-loss premiums imply greater risk to both firms and lower premiums imply less risk to both firms.

1.2 Taxonomy

In this overview, the papers have been grouped according to some common modeling approaches. Within each grouping the papers have been sorted according to the year of publication. Where one paper builds or expands on the contents of another paper placing these papers consecutively for ease of reading has taken priority over maintaining chronological ordering.

Section 2 is concerned with processes in continuous time. This appears to be the most common approach in the literature. Papers are grouped according to the type of dependency between the different processes (correlation, shared processes or thinning). Processes where the current state of the process is dependent on previous events are considered in section 3. This includes Markov chains, where only the most recent event or claim influences the next one, and also self-exciting processes where all previous events may influence the next claim. Section 4 summarizes approaches that are not based on processes over time. This includes a collection of papers with Bernoulli variables that arise from considering life insurance as well as three articles concerned with claim sizes. Section 5 groups papers that could not be categorized into any of the first three sections.



2 Modeling Based on Continuous Time Processes

Given several types of policies, these models impose dependence between the types of policies by introducing interaction between the number of claims of each type. The occurrence of claims depends on certain continuous time processes and the

models seek to determine the total number of claims of each type. In almost all cases the cost of the corresponding claims are determined independently from the time of their occurrence (and are assumed to be iid).

These modeling approaches include correlated processes, shared processes and processes with thinning.

2.1 Correlated Processes

Consider n types of claims, correlated process models assume n claim processes, one for each type. Dependency between the types of claims is caused by the correlation between the processes. This correlation is determined outside the model.

Ambagaspitiya (1998*b*) considers the total cost of all types of claims, where the number of claims of each type are correlated. The costs for an individual claim are independently and identically distributed with a known distribution depending on the type of claim. Some results can be derived in the general case. Analytic results are given for the special case where the number of claims has Poisson distributions and the cost of claims has gamma distribution.

Ambagaspitiya (1998*a*) uses the bivariate-Lagrangian Poisson (BLP) distribution to model the joint distribution of the count of two types of claims. A compound BLP distribution is introduced, this could be used to model the total cost of two types of claims. A recursive technique to represent the probability mass function of the BLP distribution, and the compound BLP distribution is given and is shown to have order $O(m^3)$.

Ambagaspitiya (1999) suggests and investigates two models. The first model has univariate distribution of claim counts and a multivariate distribution of claim costs. It is intended to represent situations where a single event results in multiple types of claims. Given a certain recursive property the distribution for total cost of claims is found. The second model has multivariate claim counts and univariate claim costs, it is intended to represent situations where different types of claims occur with some joint distribution and the cost of all types of claims has the same distribution. In special circumstances the second model is equivalent to a convolution of the first model. This shifts the emphasis from the number of claims to the cost of claims.

Vernic (1999) is interested in the distribution of the total number of claims in a bivariate model. Given that the joint counting distribution for the two claim types satisfies a particular recursive form, Vernic derives a recursive scheme for the distribution of the total number of claims (in a similar way to Vernic 1997). The use of a recursive formula removes the need to consider convolutions. The particular recursive relationship is satisfied by the binomial, negative binomial and Poisson bivariate distributions (see also Vernic 2004).

2.2 Shared Processes

This approach includes the dependency mechanism between processes as part of the model. We consider n types of claims generated by m processes, $m > n$. In general n (of the m) processes are claim process, one for each type of claim. The remaining $(m - n)$ processes are occurrence processes and result in claims of multiple types.

Vernic (1997) analyses the bivariate generalized Poisson distribution. The model uses two claim processes that generate two types of claims. Dependency is due to an occurrence process which generates simultaneous claims of both types. The probability generating function and moment generating function are derived. The joint probability for the number of claims of each type can be calculated recursively and formulas for this are given. Vernic uses the method of moments to estimate the parameters. The paper ends with two numerical examples where the estimation is fitted to existing data.

Extending the bivariate case, Vernic (2000) considers a multivariate generalized Poisson distribution. Dependency between n types of claims is due to an occurrence process which generates claims of all types. The probability and moment generating functions are given and a recursive method to calculate the probabilities follows. Two approaches for inference are suggested: the first is by method of moments, which is straightforward. The second approach uses method of moments in conjunction with the zero cell frequency method. This is slower but makes use of all available information.

Cossette & Marceau (2000) use Poisson and negative binomial models to explain the number of claims for several types of claim. The Poisson model has claim processes for every type of claim, and occurrence processes which generate simultaneous claims of two or more types of claims, for every combination of claims. As the Poisson model requires identical mean and variance, the negative binomial model is given as an alternative when the variance exceeds the mean. In the negative binomial model, the number of claims of each type is the sum of two negative binomial distributions, one unique to the type of claim and one shared with all other types of claim. The probability generating function and characteristic function are given for each model along with numerical examples for the bivariate case.

Gregory (2002) suggests a bivariate model where the shared occurrence process may be a Poisson or gamma process. The usefulness of discrete models to approximate continuous ones is emphasized and a transformation from continuous to discrete processes is given. Suggestions for inference are provided. This paper is written in the context of damage accumulation where the damage is caused by shocks. Gregory demonstrates the model using the deterioration of two types of blood pressure.

Many models incorporate homogeneous Poisson processes. Rodrigues et al. (2002) consider the union of two dependent non-homogeneous Poisson claim processes. Dependency between the two processes is created by a shared occurrence

process that generates claims of both types. The probability generating function and PDF for the resulting process is given. A special case is identified when the intensity matches a Weibull process. Rodrigues et al. estimate the parameters, in general and for the special case, using Monte Carlo simulation.

Yuen et al. (2002) consider a model where the claim processes have exponential inter-arrival times and the occurrence processes have Erlang inter-arrival times. As in Cossette & Marceau (2000) the occurrence processes result in simultaneous claims of more than one type. Analysis involved manipulating the model to consider the exponential and Erlang inter-arrival times separately. Attention is given to deriving the ruin probability and when claim sizes have exponential distribution this can be found explicitly. Asymptotic results are given in the case where the ruin probability can not be analytically determined.

The same model is further considered in Yan et al. (2006) who are concerned with the distribution of the surplus immediately before and after ruin. Formula and asymptotic results for the ruin functions are given first with their proofs following in a later section.

Ivanova & Khokhlov (2003) consider a model that allows for all possible choices of occurrence processes to generate all possible combinations of simultaneous claims (given n types of claims there will be $2^n - 1$ occurrence processes, n of which will be event processes). This is expressed using a binary multivariate index. Ivanova and Khokhlov refer to this as intersectional dependence and derive the generating function. In order to avoid double counting when determining the number of claims of each type, a transformation is proposed that enables easier summation. Possible approaches to parameter estimation are given but an explicit methodology is not provided.

Ivanova (2008) is a presentation that appears to be largely based on Ivanova & Khokhlov (2003). Ivanova suggests an approach to inference where previous periods are used to predict future time periods. This approach can be solved iteratively and a simplified form can be used when certain stationary conditions are satisfied.

2.3 Processes with Thinning

Dependency can be introduced between processes using thinning. An occurrence process generates events which result in a claim of type i with probability p_i and claims of type j with probability p_j . There is no limit on the number of claims that can result from a single event and all the claims that arise from a single event occur simultaneously. Some events always result in claims of type i ($p_i = 1$) and result in sub-claims of type j with probability (p_{ij}). Shared processes (as discussed in the previous section) can be thought of as a special case of thinning where $p_i \in \{0, 1\}$.

Yuen & Wang (2001) introduce the idea of claim thinning and provide a clear explanation of how it works. Analysis focuses on determining the number of claims of each type. The size of claims is independent of whether multiple claims arise

from a single event and independent of the occurrence processes that generated the event. Claim sizes are shown to be straightforward to include in the model when they have exponential distribution. Yuen and Wang tabulate ruin probabilities incorporating discount rates.

The interaction model proposed by Yuen & Wu (2003) extends the previous model by imposing a binomial distribution on the number of sub-claims. The expected number of claims, along with the variance and covariance of the number of claims, is then given. When the probability function satisfies a certain recursive relationship these are better behaved. It is noted that only the Poisson, binomial and negative binomial distributions satisfy the required recursive relationship.

Pfeifer & Neslehova (2004) give an overview of copulas which are then applied to construct correlated bivariate Poisson distributions. Particular interest is given to a construction that enables negative correlation between Poisson distributions. Pfeifer and Neslehova go on to consider Poisson processes. Two models are suggested both of which use copula to thin events into claims. In the first model events arise according to a single Poisson occurrence processes. In the second model events arise from multiple Poisson occurrence processes, these are linked via a copula and are hence dependent.

Wang & Yuen (2005) consider a model where types of claims are assigned to (potentially overlapping) groups and each group is subjected to events according to a Poisson occurrence process. Events for a group result in a claim of type j with probability p_j . Wang and Yuen give attention to the effect of incorrectly specifying the number of groups to include in the model. Riskiness (as measures by the Lundberg exponent) can be due to dependence in the model or due to choosing too few groups for the model.

2.4 Summary

The papers related to correlated processes are Ambagaspitiya (1998*b*), Ambagaspitiya (1998*a*), Ambagaspitiya (1999) and Vernic (1999). The papers related to shared processes are Vernic (1997), Vernic (2000), Cossette & Marceau (2000), Gregory (2002), Rodrigues et al. (2002), Yuen et al. (2002), Yan et al. (2006), Ivanova & Khokhlov (2003) and Ivanova (2008). The papers related to processes with thinning are Yuen & Wang (2001), Yuen & Wu (2003), Pfeifer & Neslehova (2004) and Wang & Yuen (2005).

3 Modeling using State Based Approaches

This section includes modeling approaches where the occurrence of the next event depends on what has already taken place. The previous section was concerned with dependency between several processes, this section also considers models where the current performance of a process depends on the past performance of the same

process.

3.1 Markov Chain Approaches

Markovian processes are those where only the present state, intensity or most recent event influences the occurrence of the next event. In order to estimate the future behavior of the process we only need to know its current state.

Oakes (1975) considers a self-exciting process where the process intensity depends on a Markov chain. For example, consider immigrants arriving in a country according to a Poisson processes. Every immigrant generates descendants according to a non-stationary Poisson process. The first generation of descendants generate a second generation of descendants according to the same non-stationary process. The instantaneous intensity of the birth process depends on the current age of all descendants and is hence a continuous Markov chain. Oakes asserts that a unique equilibrium intensity exists and derives explicit solutions for the distribution of the count of the number of events. Some of the properties for the time between events are also be derived.

Hsia (1976) is concerned with estimating the proportion of time that a three-state stochastic process spends in each state. Given the total running time for the process the proportion of time spend in each state is dependent on the proportion of time spent in each other state. Hsia constructs the joint PDF for the time spent in each state by considering the number of times the process enters a particular state and the number of combinations of the order in which different states could have been visited.

chain model for rainfall. Atmospheric data is collected by spatially clustered weather stations and hence there is dependence between the observations. The data is used to determine the current state of an underlying Markov chain (the hidden Markov chain) and the occurrence of rainfall depends solely on the current state of the Markov chain. The EM-algorithm is recommended for parameter estimation with Monte Carlo maximum likelihood (MCML) to avoid certain unwieldy functions. Even with MCML, parameter estimation is still computationally lengthy. Hughes et al. (1999) construct a non-stationary hidden Markov chain model. Observations of the system are clustered and hence there is dependence between them. The data is used to determine the current state of an underlying Markov chain (the hidden Markov chain) and the behavior of the model depends solely on the current state of the chain. The EM-algorithm is recommended for parameter estimation with Monte Carlo maximum likelihood (MCML) to avoid certain unwieldy functions. Even with MCML, parameter estimation is still computationally lengthy.

Bauerle & Grubel (2008) suggests a model where the state of a Markov chain, representing the number of claims for n types of claim, can be expresses as an n -dimensional vector. Transitions represent a claim occurring. An occurrence process determines when a transition occurs and transition probabilities are dependent on

the current state. Two models of transition probability are given for the bivariate case. The first where one type of claim effects the future transition probabilities of another type of claim. The second where one type of claim effects the future transition probabilities of the same type of claim.

3.2 Self-Exciting Processes

Self-exciting processes are those where additional claims change the likelihood of future claims. While there are similarities between self-exciting and Markovian approaches, self-exciting processes require more information about previous claims and hence lack the Markovian property.

Hawkes (1971) constructs a self-exciting point process where the arrival rate of claims is expressed as a function of the timing of all previous claims. By simple extension, the arrival rates for two types of claims are expressed as functions of the timing of all previous claims of both types. Analytic solutions exist where the effect of previous claims on the arrival rates decays exponentially. Hawkes observes that certain self-exciting processes can have identical properties to doubly stochastic processes and hence data analysis of these properties will not be able to distinguish between the two types of processes.

Inference of the parameters of Hawkes' self-exciting model are considered by Ozaki (1979). General results are provided and specific results are given for the univariate case where the effect of previous events on the arrival rate decays exponentially (a case given attention by Hawkes). Ozaki demonstrates the inference procedure on simulated data and also gives the technique for simulation.

Mino (2001) considers parameter estimation of a one-memory process, where only the most recent claim influences the arrival rate. The EM-algorithm is chosen to avoid nonlinear optimization problems that arise with maximum likelihood estimation. A continuous time approach is initially chosen and this is then discretized for ease of computation. The more similar the claim process is to a Poisson process, the less reliable the estimates are found to be. The results are supported by the use of Monte Carlo simulations.

Giesecke & Goldberg (2005) are concerned with predicting claims in the context where an initial claim can trigger additional claims. A self-exciting process with random thinning is used to model the occurrence of claims. Giesecke and Goldberg also consider a doubly stochastic process. Due to complexity, both approaches are simplified by a 'compensator function' which has the same intensity as the complete models but which handles the clustering of events in an analytically tractable way.

3.3 Summary

The papers related to Markov chain approaches are Oakes (1975), Hsia (1976), Hughes et al. (1999) and Bauerle & Grubel (2008). The papers related to self-exciting processes are Hawkes (1971), Ozaki (1979), Mino (2001) and Giesecke & Goldberg (2005).

4 Non-Process Based Models

This section differs from the previous two sections as it does not focus on estimating the number of claims that arise from continuous time processes. This section includes models where claims arise from Bernoulli trials, and models in which the cost of claims are not assumed to be independent and identically distributed.

4.1 Approaches with Bernoulli Trials

These models introduce dependency between a known number of Bernoulli trials. This will be discussed in the context of life insurance where, in a given time period, there will be a single claim against a policy if the policy holder dies. The timing of claims in a given time period is ignored. These models might be extended by considering several consecutive time periods and allowing for dependency between Bernoulli trials across time periods.

Dhaene & Goovaerts (1997) consider dependency between life insurance policies. The lifetimes of couples are known to be positively correlated. It is shown that assuming independence between policies underestimates the risk to the insurer (as measured by stop-loss premiums) if the true situation with dependence can be modeled using a two-point distribution. However this result does not generalize for three-point (or more) distributions. Dhaene and Goovaerts also consider dependencies between all policies, not just pairs of policies. Dependence where no claim on a policy implies no claims on all policies with lower risk is shown to give rise to the greatest risk to the insurer.

Bauerle & Muller (1998) suggest two models for dependency between a collection of insurance policies. The first model groups policies and determines their risk based on global factors (those that affect all policies), group factors (those that affect all policies in a group) and individual factors (those that only affect a single policy). It can be shown that fewer groups and larger group sizes implies greater risk to the insurer. The second model constructs a list of external mechanisms and models the risk of each policy based on the number of mechanisms that may affect it. Four different types or ordering are described: stochastic, stop-loss, super-modular and symmetric super-modular orderings. These are used to determine which policies are riskier than others.

Cossette et al. (2002) consider a special case of the first model suggested by Bauerle & Muller (1998) and derive the moment generating function, variance and covariance for the total cost of claims in this model. This is compared to a second model that uses copula, with attention given to the Cook-Johnson and Gumbel copula. Comparison of these models suggests that the use of copula allows the second model to incorporate greater dependence. Examples of both models are illustrated with varying parameters.

Genest et al. (2003) are interested in modeling dependence between insurance policies or groups of insurance policies using copula. The Clayton model, the Gumbel family and the Frank family of copula are recognized as popular choices for actuarial applications. These are all encompassed by a three-parameter family of Archimedean copula. Genest et al. suggest Poisson approximations for single and multi-class Archimedean models and show that these approximations are able to introduce heterogeneity between groups of policies.

Ribas et al. (2003) consider correlated pairs of life insurance policies (for example husband and wife). Given the correlation between policies two recursive models are suggested to calculate the distribution function for the total cost of claims. The first model is computationally faster but requires all pairs of dependent policies to have the same correlation. The second model permits any choice of correlation between pairs of dependent policies but is computationally slower. Parameter estimation is likely to be easier in the first model.

4.2 Models of Claim Sizes

A common assumption in many papers is that claim sizes are independent and identically distributed, and further more, are independent of the number of claims. These papers are concerned with dependent claim sizes.

Frees & Valdez (2008) construct a hierarchical model of insurance claims. The model includes claim frequency, claim type and claim cost. Claim frequency is determined first and is fitted with Poisson or negative binomial models, with or without random effects. A single event may result in multiple claim types. Claim types are modeled using a multinomial logit model. Claim costs are determined conditional on the claim frequency and claim type(s). Dependency between the cost of multiple claim types arising from the same event is introduced using copula. A heavy tailed distribution is preferred for the cost of claims, Frees & Valdez suggest the generalized beta distribution of the second kind.

Kolev & Paiva (2008) consider two models where the costs of a given number of claims are correlated. In the first model all pairs of claim costs have the same correlation and in the second model all pairs of consecutive claim costs have the same correlation. For simplicity this is reduced to a Bernoulli model where the cost of each claim is replaced by an indicator for whether the cost of the claim exceeds a known threshold or not (we could construct a full distribution by considering a

collection of thresholds). For the second model with adjacent correlation the probability generating function can be derived by considering transition probabilities from one claim to another, in a similar way to Markov chains.

Meng et al. (2008) consider a model where the cost of each claim is dependent on the time since the previous claim. When events occur frequently people adapt and are prepared for the next event, so the claims from an event are small, but if events occur infrequently people become complacent and are unprepared for the next event, so the claims from an event are large (consider that earthquakes occur frequently in Japan so the population has adjusted and the cost of claims from earthquakes are small). The model compares the inter-event time to a threshold. If the time is within the threshold the cost follows a distribution with a small expected value. If the time is beyond the threshold the cost follows a distribution with a large expected value. Meng et al. derive analytic results for specific distributions of inter-event time, threshold and claim costs.

4.3 Summary

The papers related to Bernoulli approaches are Dhaene & Goovaerts (1997), Bauerle & Muller (1998), Cossette et al. (2002), Genest et al. (2003) and Ribas et al. (2003). The papers considering models of claim sizes are Frees & Valdez (2008), Kolev & Paiva (2008) and Meng et al. (2008).

5 Other Approaches

This section summarizes research that contributes to our understanding dependency between random variables and could be useful in modeling insurance claims.

5.1 Classifying Dependency

These papers consider dependent multivariate variables and suggest methods of classifying or ordering the severity of the dependence.

Denuit et al. (2002) introduce a measure of probabilistic distance. This is a quantitative measure for the effect of dependence between two variables and a method to calculate this distance is presented. The measure is demonstrated across nine different types of examples of dependence, including: copula, mixtures, shared Bernoulli processes and epidemic models.

Ostrovskaya (2006) classifies the relationships between random variables as uncorrelated, convolutionally independent or independent and suggests a measure for quantifying these relationships. Using this quantified measure, it is shown that there exists a threshold that separates uncorrelated and convolutionally independent relationships. Independent relationships occur as the limiting case of convolutionally independent relationships.

5.2 Multivariate Distributions

These papers consider multivariate distributions (and in particular multivariate phase type distributions) and where possible derive their properties. Multivariate distributions are a natural way of incorporating dependence between random variables. Phase type distributions are used for their ability to represent multivariate distributions. The performance of phase type distributions may be easier to understand than the multivariate distributions they represent.

Goff (2001) recognizes that many multivariate stochastic models have distributions that can be modeled by phase-type distributions (for example, the model by Marshall & Olkin 1979). Goff focuses on multivariate discrete phase-type distributions and proposes a simulation method to estimate the multivariate distribution. Two types of dependence in the multivariate distribution are noted: dependence caused by covariance and dependence caused by one variable bounding another. Four questions and three applications for further study are proposed.

Ivanova & Khokhlov (2001) wish to reconstruct the joint multivariate distribution given its marginal distributions. They recognize that, in general, such solutions are non-unique. Given the natural exponential families are appropriate and the marginal distributions are Poisson it is possible to construct the unique multivariate distribution. The reconstruction generates additional parameters, the values of which must be later solved for.

The presentation by Frostig (2005) proposes using a subgroup of phase-type distributions to model the times between insurance claims. Frostig is interested in determining the probability of ruin and the expected deficit at ruin. It has been shown that for exponential inter-arrival times the n^{th} claim has a phase-type distribution. Under specified conditions Frostig derives some analytic results.

Vernic (2005) looks at the skew-normal distribution and proves results for the multivariate case. The skew-normal distribution can be defined as a normal distribution truncated to be above some lower bound. Vernic defines the scale mixture of a skew-normal distribution and derives some of its properties. Several special cases of the scale-mixture are highlighted at the end.

5.3 Approximating Distributions

The relationships between dependent variables are frequently too complex to be resolved analytically. These papers consider alternative approaches, including finding alternative tractable functions, discretization and numerical methods.

Campana et al. (2000) wish to obtain a good approximation for a class of renewal processes defined by a particular differential equation. Multiple estimation approaches are considered including: exponential curves, Volterra integral equations and Laplace transforms, and Neumann's series. In the limiting case the log-gamma and Pareto distributions are considered sufficient to approximate certain heavy tailed distributions.

Vernic (2002) looks at applications of results from Goovaerts et al. (2004) that estimate the tail probability of Pareto-like distributions. Of particular interest is the value of a randomly weighted sum, where the weights are intended to represent discount factors.

Vernic (2003) deals with the arithmetization of univariate and bivariate continuous distributions. This is a means of making continuous distributions discrete so they can be more easily manipulated in certain contexts (for example: integrals can be replaced with summations). After specifying how to arithmetize a distribution, Vernic defines a measure of distance between the distribution and its arithmetized form. The distance gives an idea of how well the arithmetized distribution approximates the continuous distribution. Three examples are used to demonstrate this measure.

Enachescu & Vernic (2005) are interested in approximating the distribution of a randomly weighted sum. Two approaches are proposed to do this, a multilayer perceptions model and a kernel model. Of these the multilayer perceptions model appears to be superior and even behaves well for small sample sizes. While the results hold in general the intended application is for the weights to be discounting factors (as in Vernic 2002).

5.4 Summary

The papers related to classifying dependency are Denuit et al. (2002) and Ostrovska (2006). The papers related to multivariate distributions are Goff (2001), Ivanova & Khokhlov (2001), Frostig (2005) and Vernic (2005). The papers related to approximating distributions are Campana et al. (2000), Vernic (2002), Vernic (2003) and Enachescu & Vernic (2005).

6 Discussion

It is common to model the arrival of insurance claims using stochastic processes over time. This enables the frequency of claims to be considered. Dependency between different types of claims is frequently imposed by dependency between the number of claims of each type. The cost of claims is often assumed to be independent of their occurrence, and to have independent and identical distributions.

This review includes very little literature regarding the distribution of claim sizes. Several papers (Ambagaspitiya 1998*b*, Yuen & Wang 2001, Yuen et al. 2002, notably) have assumed exponential or gamma distributed claim sizes in order to provide analytic results. As the distribution of claims may depend on the type of insurance offered (for example: claim sizes for health insurance and fire insurance may have different distributions) requiring a particular distribution may limit the applications of a model.

Next we present several possible directions for future research.

It would be useful to develop a flexible model where the size of claims may effect the frequency of claims and where the frequency of claims may effect the size of claims. To make the model analytically tractable it may be desirable for the model to have a long run equilibrium at which claim frequency and claim costs are independent, and allow claims to cause short term deviations away from this equilibrium.

It may be worthwhile investigating models for damage accumulation in reliability analysis. As time passes and a system accumulates damage it will become more likely to fail. In a similar way, as time passes and as claims are made, future claims on the same hazard event become less likely. So we equate system failure, in reliability analysis, with the final claims relating to a particular hazard event, in insurance modeling.

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