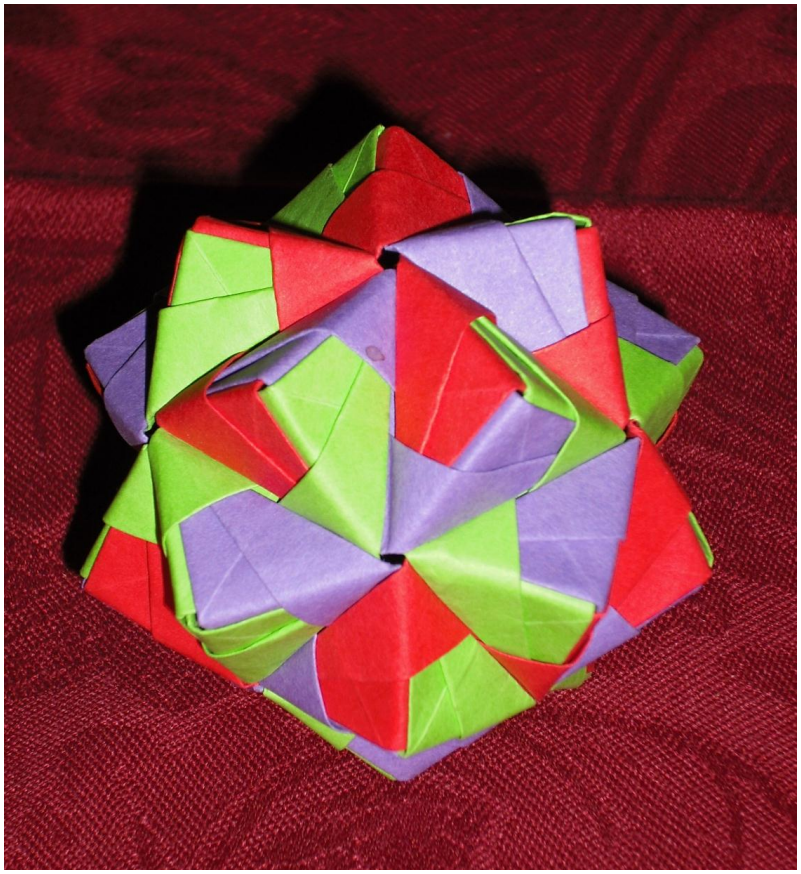
Mathematical Art

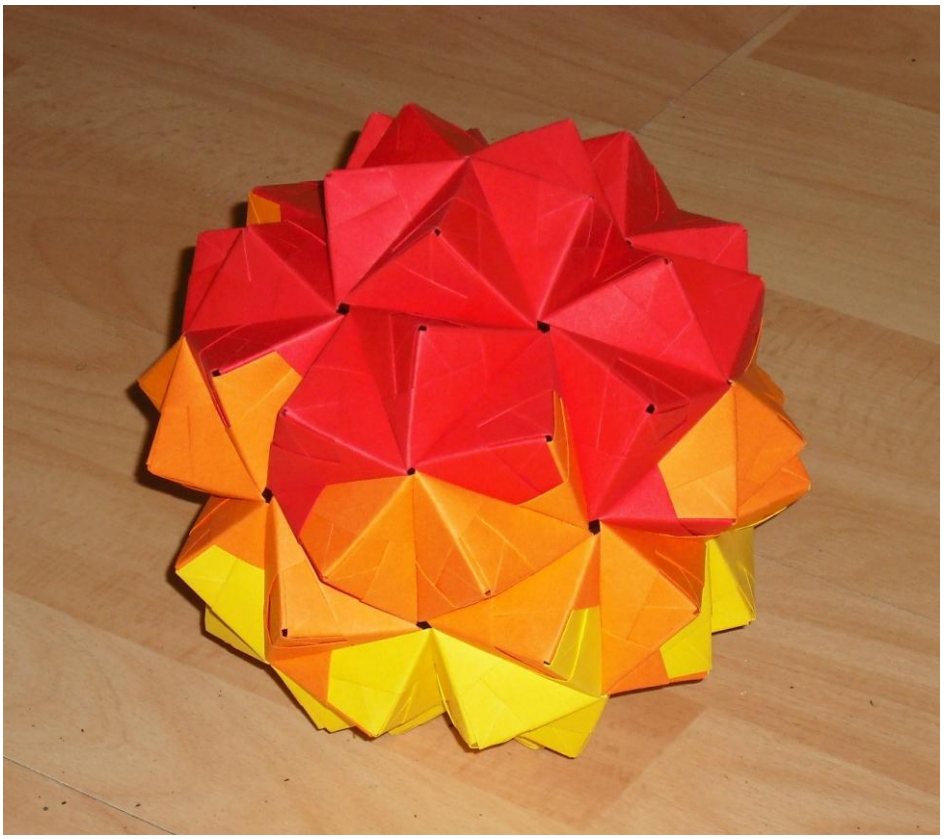
Many of our students have asked how I made the shapes in the display cabinet and the Halloween pumpkin. I didn't think up any of the ideas from scratch, I found instructions for things on the internet and in books. Some of my shapes just follow these instructions and some are modifications that I thought of (although I am sure that other people have thought of these things too).

The origami Halloween pumpkin and a few others are based on the instructions from

http://nuwen.net/poly.html

but I use bigger squares of paper.

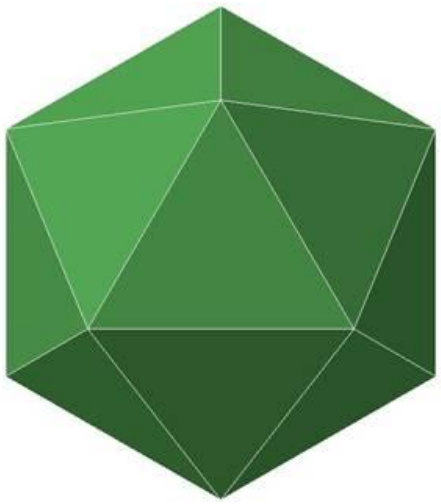




Note that the guy who wrote this website describes his "Epcot Ball" in terms of the hexagons and pentagons of a soccer ball (AKA Bucky ball, AKA truncated icosahedron). There is another way to describe it in terms of an **icosahedron.**

What is an icosahedron?

- Have you seen a 20 sided dice? That shape is called an icosahedron.
- Each **face** of an icosahedron is an equilateral triangle, there are twenty faces, and you can see from the picture below that at the corner of every triangular face is a point where five triangles meet. That is the repeated pattern of an icosahedron – at every corner point, **five** triangles meet.

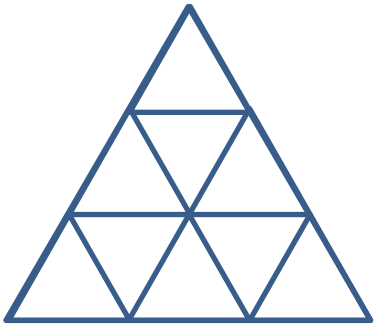


An Icosahedron

[I found this picture by searching google images for "icosahedron."]
Picture from http://www.kjmaclean.com/Geometry/Icosahedron_files/image002.jpg

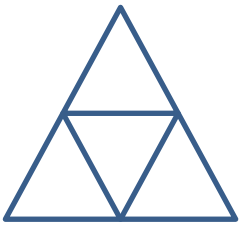
Back to the origami.

How can you describe an Epcot ball in terms of an icosahedron? Firstly, each of the origami "pyramids" (see website above) is triangular and its base is an equilateral triangle. Secondly, note that on the website, the icosahedral ball using 30 pieces is constructed so that the base of each pyramid is a face of the underlying icosahedron. For the Epcot ball, take each face of an icosahedron and construct it from nine pyramids as in the following diagram:

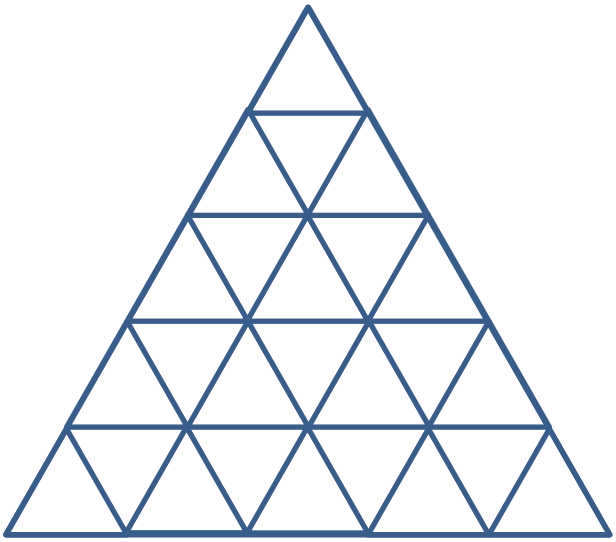


*[Note, given that the Epcot ball has 9 times as many pyramids as the basic icosahedral ball, it requires nine times as many pieces of paper, namely 30*9=270 pieces.]*

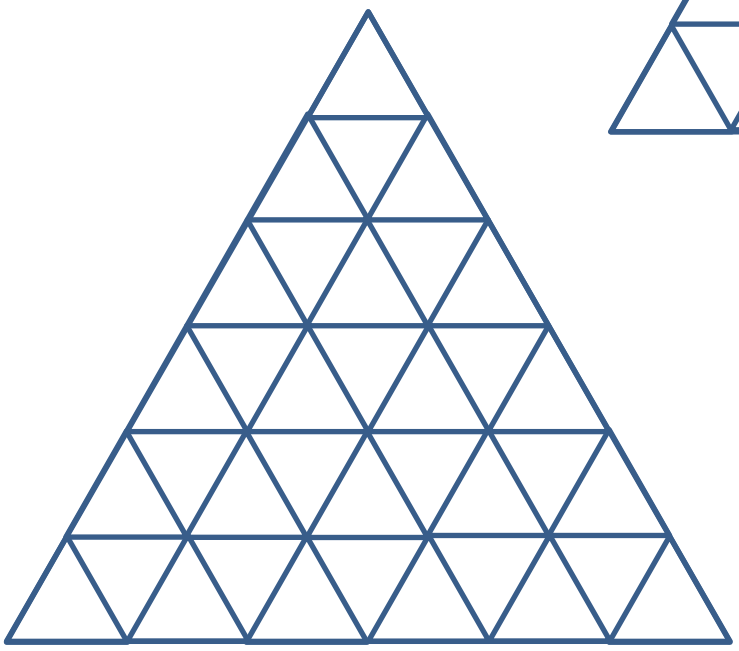
Of course, you can choose to break up the faces of the icosahedron into other numbers of pyramids, it doesn't have to be nine:



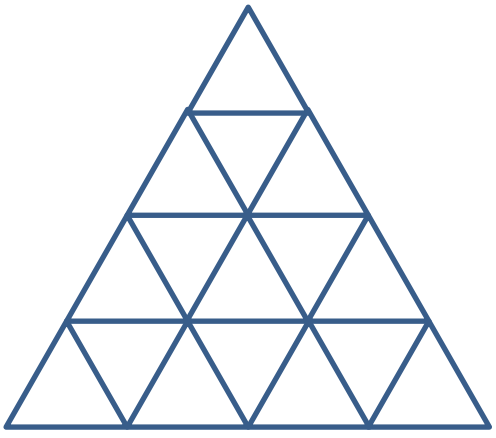
Four pyramids
30×4=120 pieces of paper



25 pyramids
30×25=750 pieces of paper

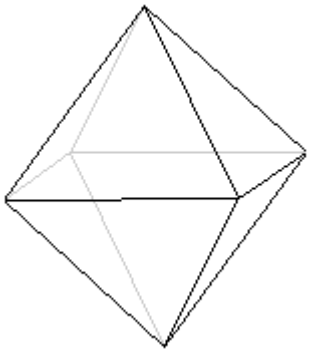


36 pyramids
30×36=1080 pieces of paper



16 pyramids
30×16=480 pieces of paper

Note also that you don't have to build an **icosahedron** with these origami pieces, you could build an **octahedron** which has just eight faces.



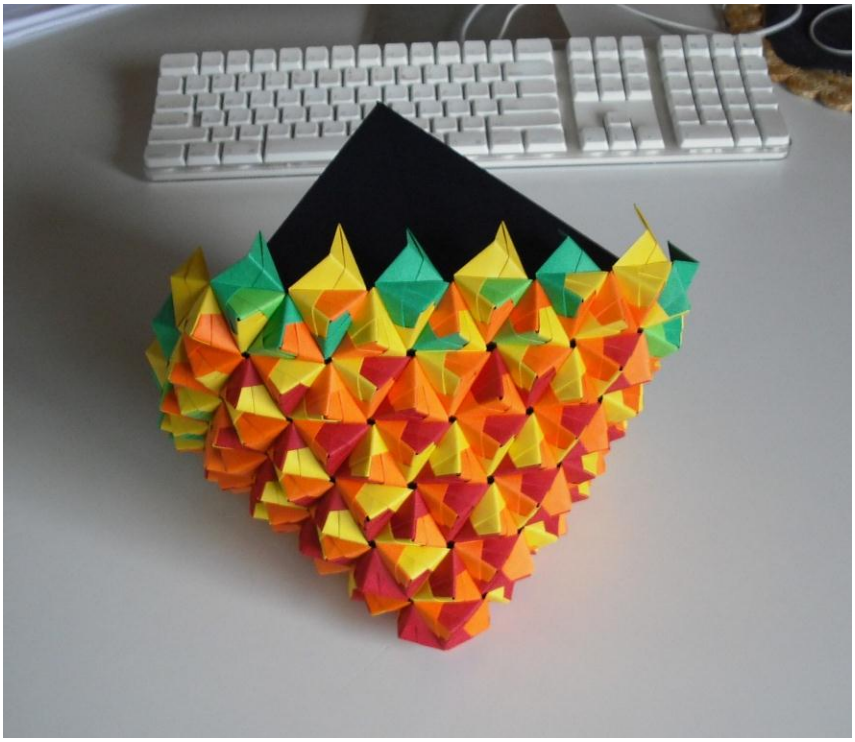
An octahedron

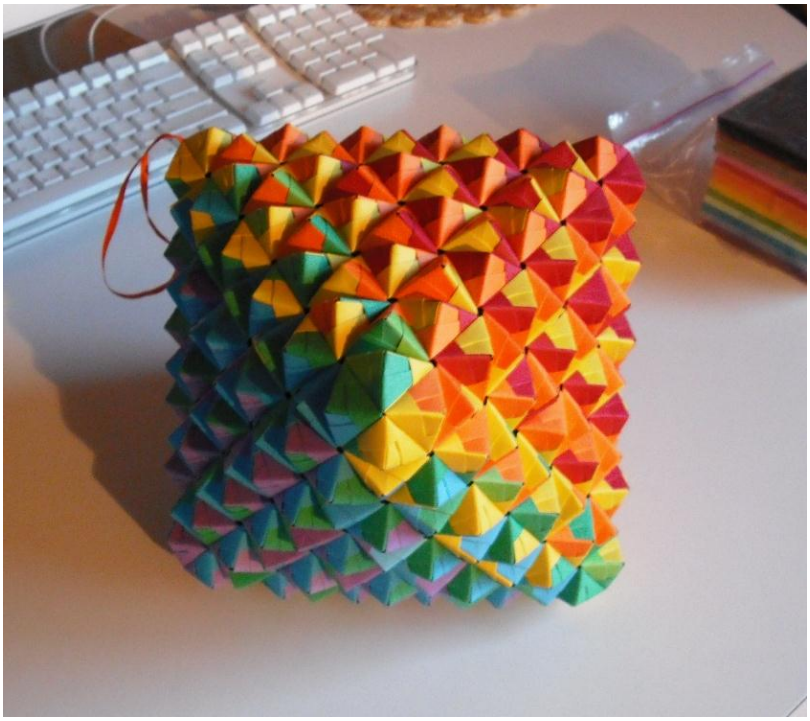
(Picture from http://www.fastgeometry.com/Encyclopedia/PlatonicSolids.htm

[In an octahedron, at the corners of the triangular faces, the pattern is that four triangles meet (instead of the five triangles of the icosahedron).]

I built an octahedron where each face was made from 36 pyramids (see photos below). The total number of pieces for the basic origami octahedral ball is 12, so when there were 36 pyramids per face, I needed 36*12=432 pieces of paper.

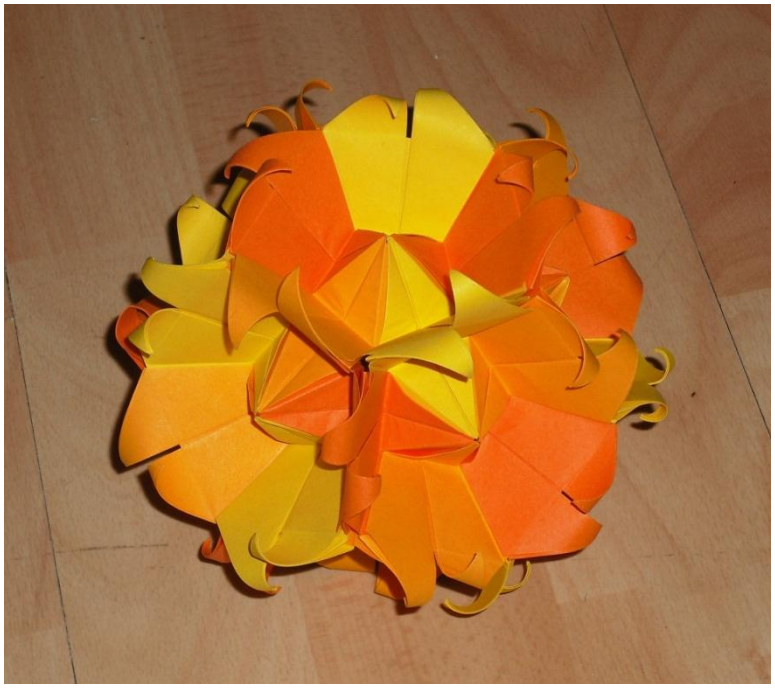
Note also that when you start building these larger structures, where each face is constructed from more than one pyramid, the structure of the origami ball is less rigid and more likely to come apart. To remedy this, I build a hollow cardboard icosahedron/octahedron of the appropriate size to sit inside the origami pieces, then I weave together the origami pieces over the top – yes that does make me a great big origami-cheat! See pictures below (the hollow cardboard octahedron is black).

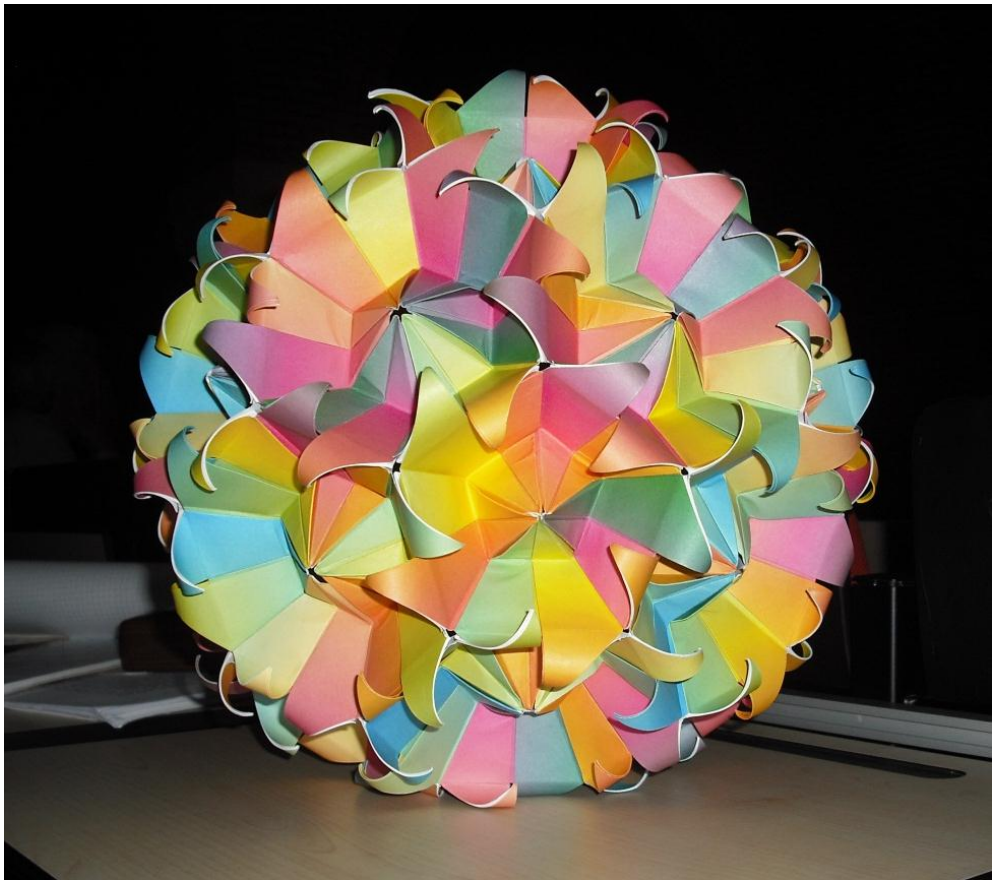


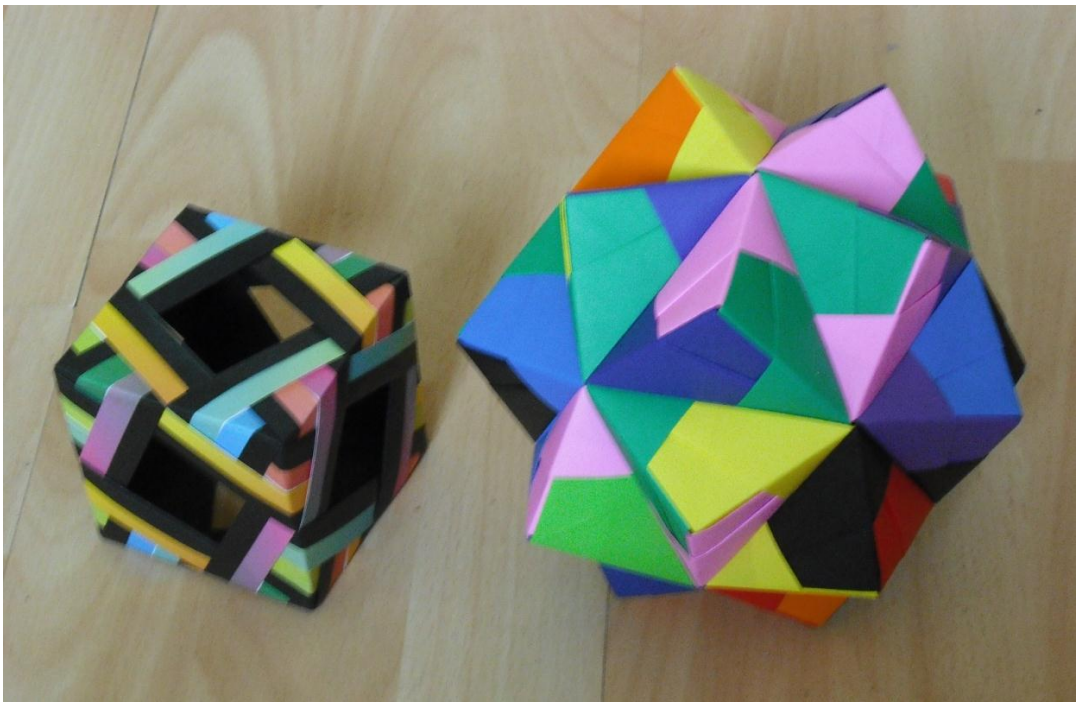


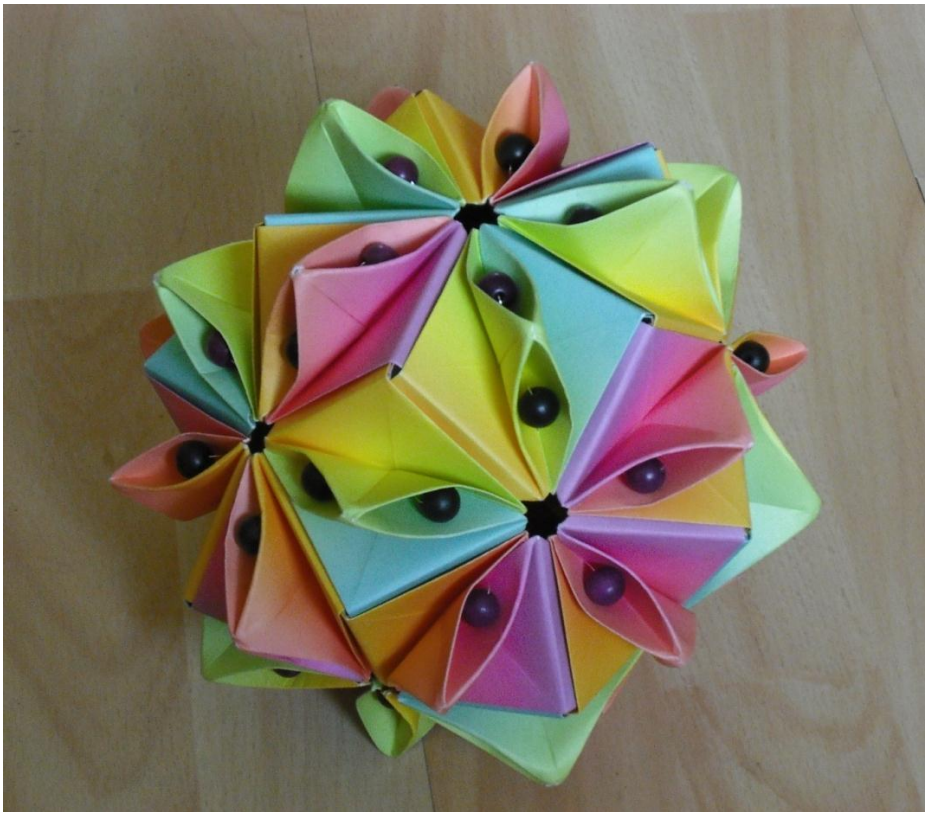
I have also obtained other origami recipes from two books I bought from amazon:

- "*The Beginner's Book of Modular Origami Polyhedra: The Platonic Solids*" by Rona Gurkewitz and Bennett Arnstein,
- "*Exquisite Modular Origami*" by Meenakshi Mukerji.



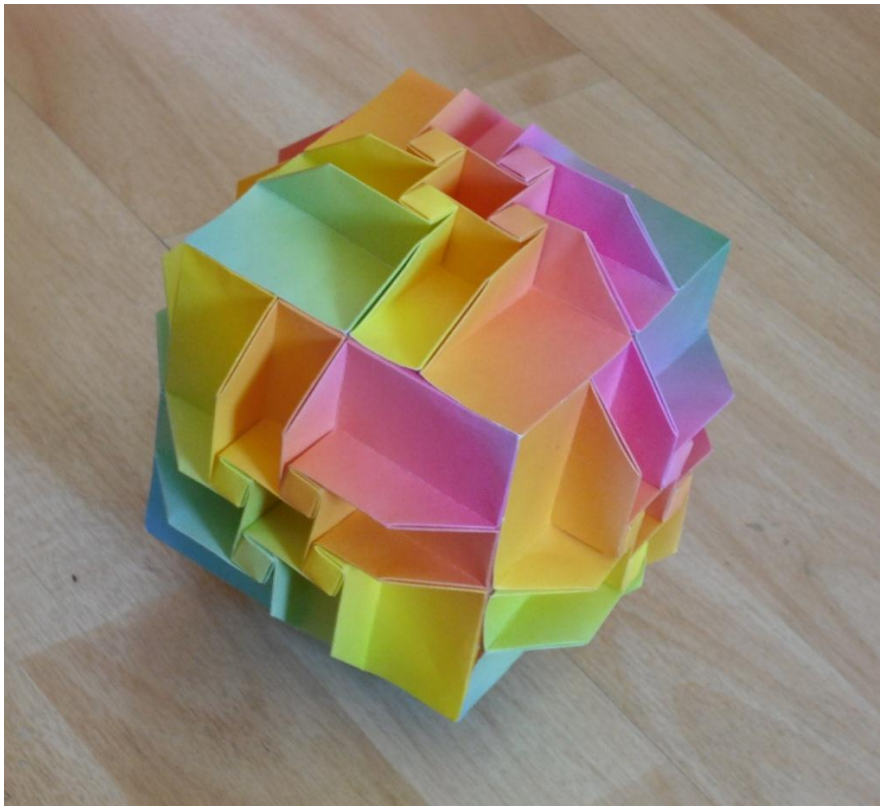






[Decorative beads added to the origami pieces.]

I have also found some cool stuff from googling “origami Christmas decoration”, “origami tessellation”, “origami polyhedron”, etc.

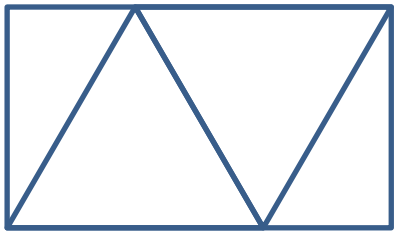


How do you build a hollow cardboard icosahedron/octahedron?

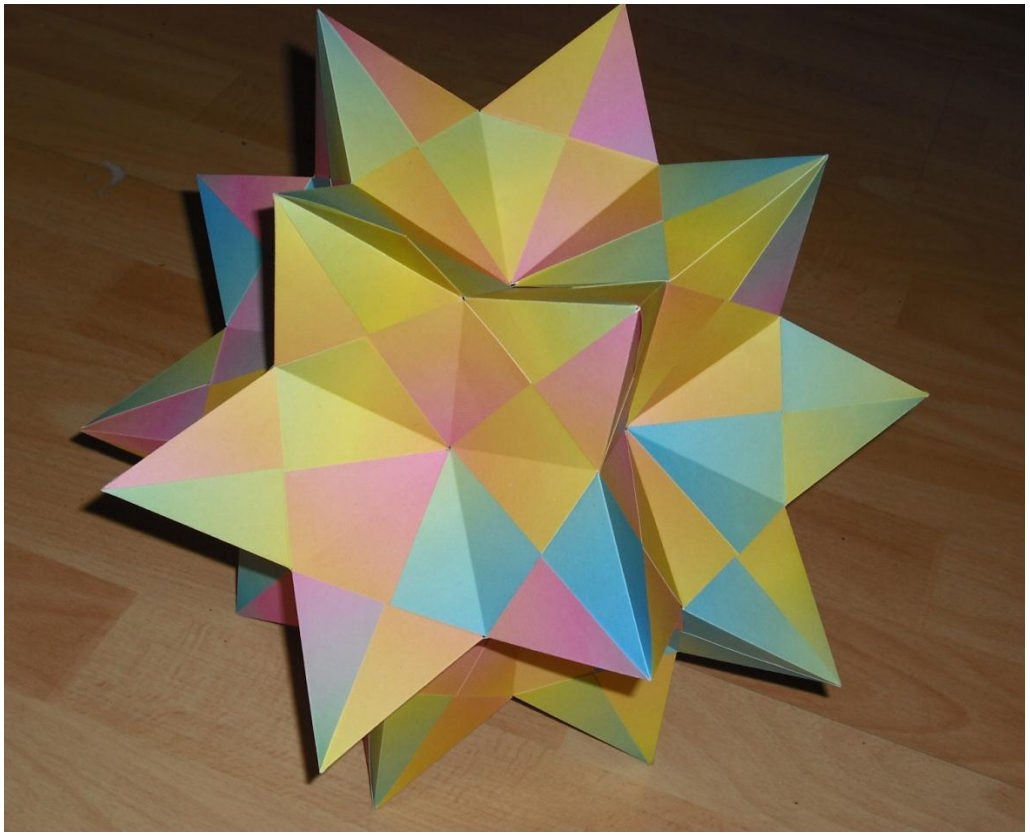
The following website tells you how to build various polyhedra from American-sized cardboard business cards:

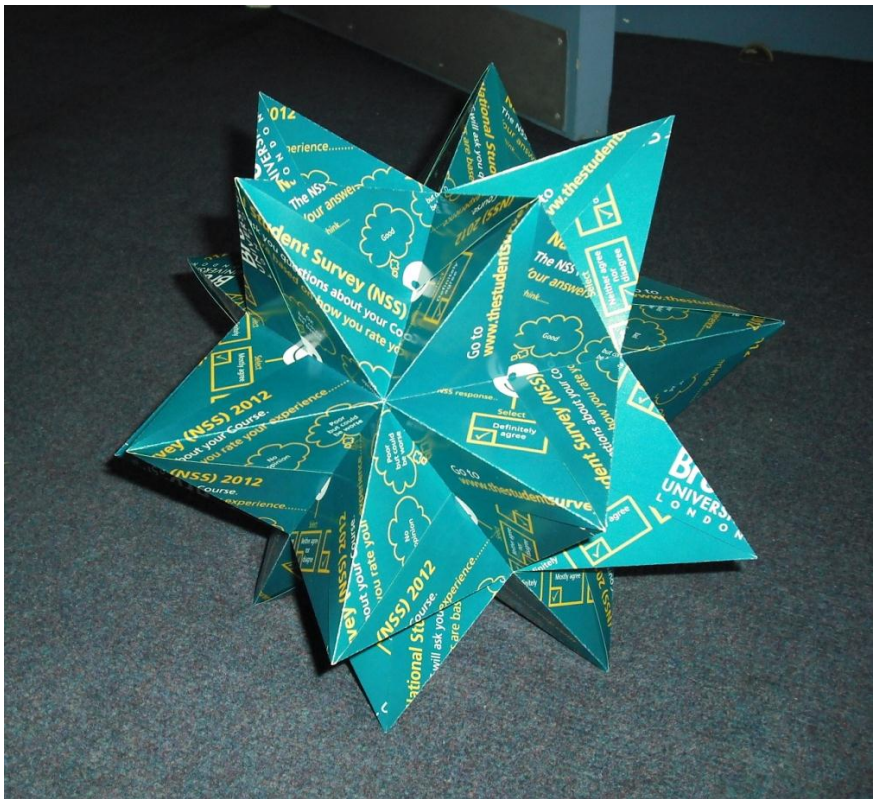
http://www.davekoelle.com/polyhedra.html

Of course, you don't have to use actual business cards. You can cut your own coloured card to whatever size you want. The key is that the rectangles need to have a specific ratio of long side length to short side length. Namely, the long side should be 1.73 times the length of the short side (1.73 is approximately the square root of 3). This is so that the three folds you make in the card will give you two equilateral triangles:



Here are a few things I have made from cardboard rectangles of these length-width proportions, and folded this way:

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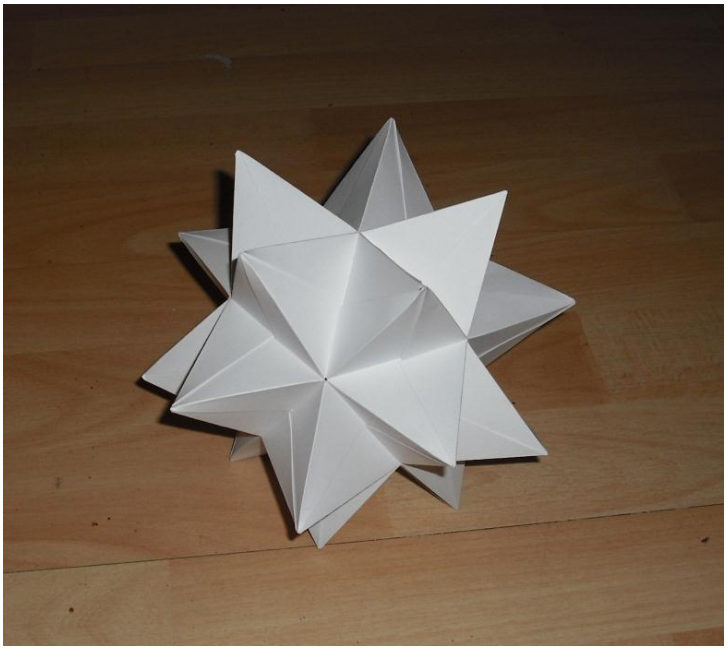
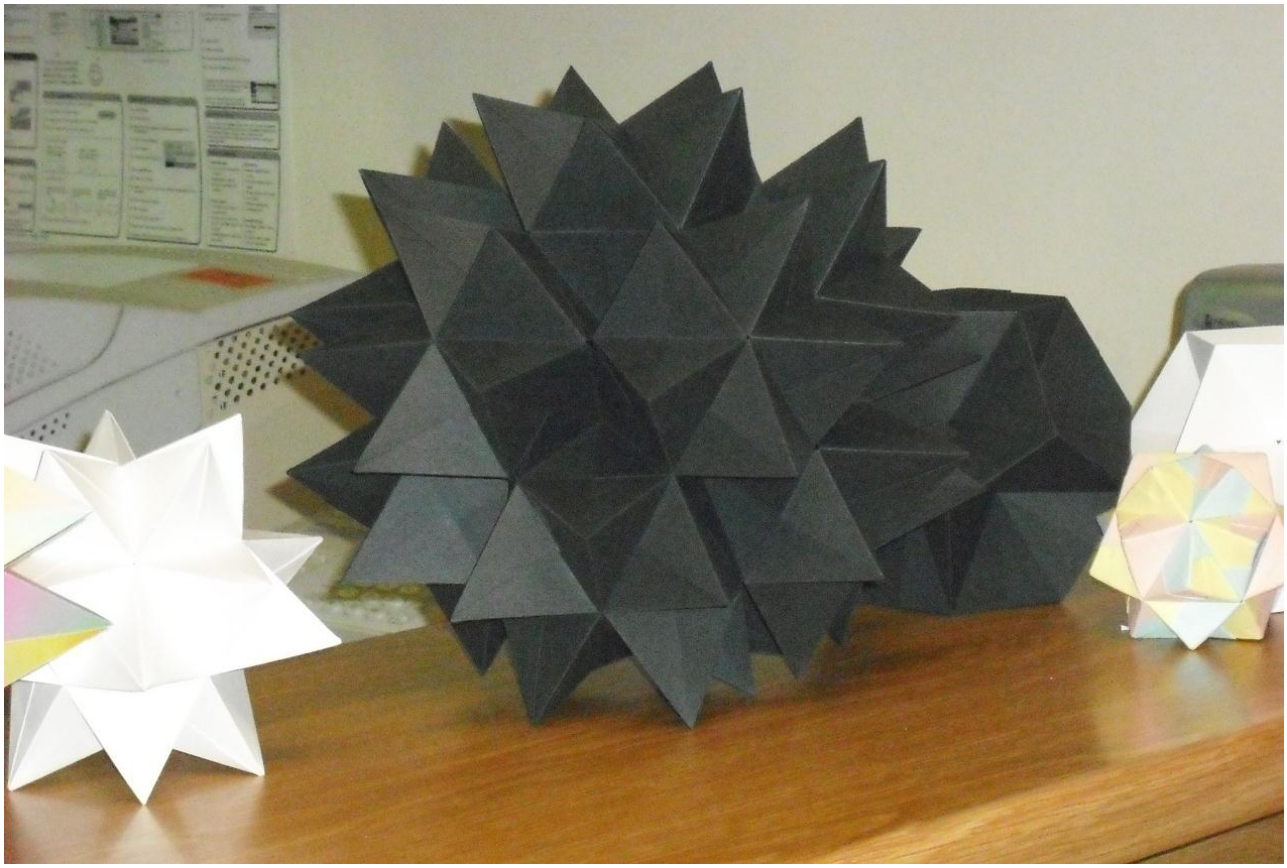
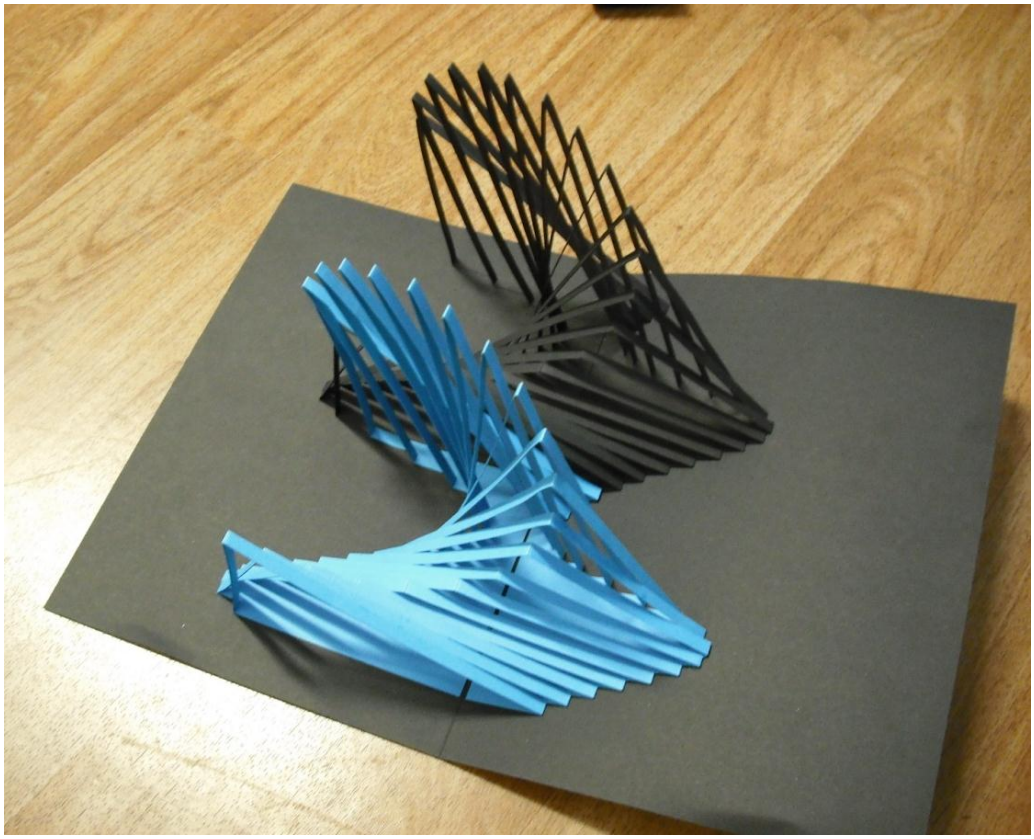




Here is a pop-up card. I don't know the web address, but I got the idea from a youtube clip. I wanted to make it so that the top corners of the pop-up bits form a spiral/helix so I did the calculations myself to achieve that effect:



Have you actually read this far? Amazing!
--Rhiannon.