# Aspects of Supersymmetry Breaking

Matt Visser

Ph.D. Thesis

*University of California at Berkeley*

September 1984

**Abstract**

We discuss N=1 supergravity coupled to gauged chiral matter. We retain noncanonical kinetic energy terms for both matter and gauge fields. The tree level spontaneous breaking of supergravity in such theories is investigated. Emphasis is placed on general results rather than any particular model.

The tree level mass matrices are calculated, and used to derive a $(mass)^2$ sum rule that retains the effects of the noncanonical kinetic energies. Even in the presence of noncanonical kinetic energies it is shown that under not too restrictive conditions we can relate the masses of leptons and quarks to the masses of their scalar partners by

$$m_0^{\pm} = \left| m_{3/2} \pm m_{1/2} \right| .$$

Attention is also drawn to the crucial role played by the analyticity of the superpotential at the origin of field space.

# Chapter 1

# Introduction

If supergravity has anything at all to do with the real world, then certainly it is a broken symmetry. Explicit symmetry breaking is inelegant and teaches us very little. On the other hand , if the underlying real theory is an extended (N>1) supergravity, the spontaneous breaking of the extended supergravity may take the theory through a N=1 symmetric phase. For these reasons the study of N=1 supergravity coupled to gauged chiral matter is interesting in that it provides a framework that may be relevant to supergravity theories in general.

Now N=1 supergravity coupled to matter is a nonrenormalizable theory [1,2]. Thus radiative corrections should not be taken particularly seriously. Perhaps the best viewpoint to adopt is that N=1 supergravity is a low energy effective theory engendered by some as yet not understood microstructure. Note that low energy in this case means $E < m_P \approx 10^{19}$ GeV. If we adopt this viewpoint then N=1 supergravity is not to be thought of as a fundamental theory, rather its status is similar to that of the non-linear sigma model for pions. In particular there is no justification for enforcing canonical kinetic energy terms in the Lagrangian. If nothing else we would expect noncanonical kinetic energies to be generated by radiative corrections in the underlying true theory. (For similar comments see [3]).

For the reasons discussed above, all comments made in this thesis will apply at tree level only. In particular, I shall discuss the construction of acceptable vacua using tree level symmetry breaking only (for alternatives see [4]). Breaking the supergravity in an acceptable way is not trivial. Even if one succeeds in breaking the supergravity itself it is distressingly easy to generate multiple vacua. The extra unwanted vacua commonly possess negative cosmological constant [5], or they may fail to break the gauge symmetry [6]; the extra vacua may even be degenerate with the phenomenologically desired vacuum [6]. At the very least the following must be satisfied:

- 1) supergravity must be broken ($m_{3/2} > 0$).
- 2) The cosmological constant must be zero ($\Lambda = V|_{vacuum} = 0$).
- 3) The gauge symmetry must be broken.
- 4) Higgsinos and gauginos should be massive.

In addition it is very desirable that:

- 5) The vacuum occurs at the unique absolute minimum of the scalar potential V.

Many models have been constructed that violate condition 5 (*e.g.* [7]), these models then have to deal with the problem of the decay of the false vacuum, a problem that I shall eliminate by fiat by imposing condition 5.

In this thesis I shall discuss general theorems indicating when these conditions may be satisfied at tree level. In addition an exhaustive discussion of mass matrices and sum rules is presented. The $(mass)^2$ sum rule of Cremmer *et al.* [2] is generalized to include the effect of noncanonical kinetic energy terms. The leptoquark sum rule of Cremmer *et al.* [6] is shown to be insensitive to the occurrence of noncanonical kinetic energies. The effect of nonanalyticity in the superpotential is also discussed.

The main tool used in this analysis is the component Lagrangian for N=1 supergravity as constructed by Bagger [1], Witten and Bagger [1], and Cremmer *et al.* [2]. These papers differ in that Bagger shows how to gauge symmetries that are realized in a nonlinear fashion. Also Bagger uses a notation that is vastly superior in that it makes manifest the geometrical structure of the various terms appearing in the Lagrangian. On the other hand, the work of Cremmer *et al.* uses only linearly realized gauge symmetries but allows noncanonical kinetic energies for the gauge bosons. Cremmer *et al.* also calculate the $(mass)^2$ sum rule, which is not done by Bagger.

For this thesis I shall be using the Lagrangian of Cremmer *et al.* [2], but the notation will essentially be that of Bagger [1]. Extensive use will be made of the geometry of Kähler manifolds and in particular of the concept of the Kähler covariant derivative. This will unfortunately necessitate a short chapter reviewing Kähler geometry. If one had attempted to use the notation of Cremmer *et al.* [2], the retention of noncanonical kinetic energies would quickly lead to calculations so cumbersome as to be prohibitive.

# Chapter 2

# Kähler Manifolds

In N=1 supergravity coupled to matter the self-interactions of the scalar fields are described by a generalized nonlinear sigma model where the manifold of scalar field values is a Kähler manifold [1,2].

The geometry of Kähler manifolds is well understood, easily accessible references are the books of Goldberg [8] and Flaherty [9]. A Kähler manifold $\mathcal{M}$ is a complex manifold whose geometry is specified by a real valued function, the Kähler potential $K$. Complex coordinates on the manifold will be denoted by $\phi^i$, their complex conjugates are denoted by $\phi^{i^*}$. The metric tensor of a Kähler manifold is given by:

$$g_{ij^*} = \frac{\partial^2}{\partial \phi^i \partial \phi^{j^*}} K(\phi, \overline{\phi}) = \partial_i \partial_{j^*} K. \tag{2.1}$$

The metric tensor is thus automatically Hermitian. By assumption the metric shall be taken to be positive definite on $\mathcal{M}$. Indices may be raised and lowered using the metric and its inverse:

$$g^{ij^*} = (g_{ij^*})^{-1} \tag{2.2}$$

for example:

$$X^i = g^{ij^*} X_{j^*}; \quad Y^{j^*} = g^{ij^*} Y_i; \quad X_{i^*} = g_{ji^*} X^j; \quad etc. \tag{2.3}$$

Note in particular that:

$$X^i Y_i = X_{i^*} Y^{i^*} = (X_i Y^i)^* \not\equiv X_i Y^i \tag{2.4}$$

Christoffel symbols may be calculated in the usual manner. Because of the Kähler structure the expressions simplify radically.

$$\Gamma \quad \begin{cases} \Gamma^i{}_{jk} = g^{im^*}\partial_k(g^{jm^*}) = (g^{im^*}\partial_{m^*})\partial_i\partial_j K. \\ \\ \Gamma^{i^*}{}_{j^*k^*} = g^{mi^*}\partial_{k^*}(g_{mj^*}) = (g^{mi^*}\partial_m)\partial_{j^*}\partial_{k^*} K = (\Gamma^i{}_{jk})^*. \\ \\ \text{all other components are zero.} \end{cases} \tag{2.5}$$

Kähler covariant derivatives will be denoted by $\delta$; we have

$$\delta_i X^j = \partial_i X^j + \Gamma^j{}_{mi}\, X^m, \tag{2.6}$$
$$\delta_i X^{j^*} = \partial_i X^{j^*}, \tag{2.7}$$
$$\delta_i X_j = \partial_i X_j - \Gamma^m{}_{ji}\, X_m, \tag{2.8}$$
$$\delta_i X_{j^*} = \partial_i X_{j^*}. \tag{2.9}$$

Naturally the covariant derivative has been chosen so that the metric is covariantly constant

$$\delta_i g_{jk^*} = 0 = \delta_{i^*} g_{jk^*}. \tag{2.10}$$

The expression for the Riemann tensor simplifies to

$$R^i{}_{jlm^*} = \partial_{m^*}\Gamma^i{}_{jl} = g^{ik^*}(\partial_l\partial_{m*} g_{jk^*}) + (\partial_{m^*} g^{ik^*})\,(\partial_l g_{jk^*}). \tag{2.11}$$

The only nonzero components of the Riemann tensor are

$$R^i{}_{jlm^*}; \quad R^i{}_{jl^*m}; \quad R^{i^*}{}_{j^*l^*m}; \quad \text{and} \quad R^{i^*}{}_{j^*lm^*}. \tag{2.12}$$

In fully covariant form

$$R_{ij^*lm^*} = -\partial_l\partial_{m^*} g_{ij^*} + (\partial_l g_{ik^*}) g^{kk^*}(\partial_{m^*} g_{kj*}). \tag{2.13}$$

The only nonzero components of the fully covariant Riemann tensor are

$$R_{ij^*kl^*}; \quad R_{ij^*k^*l}; \quad R_{i^*jkl^*}; \quad \text{and} \quad R_{i^*jk^*l}. \tag{2.14}$$

The Riemann tensor possesses the symmetry

$$R^i{}_{jkl^*} = +R^i{}_{kjl^*} = -R^i{}_{jl^*k}. \tag{2.15}$$

The Ricci tensor may be defined by

$$R_{ij^*} = g^{kl^*} R_{il^*kj^*} = g^{kl^*} R_{ij^*kl^*} = g^{kl^*} R_{kl^*ij^*}. \tag{2.16}$$

Observe that the Ricci tensor is Hermitian. The contracted Bianchi identities read

$$\partial_k R_{ij^*} = \partial_i R_{kj^*}, \tag{2.17}$$

$$\partial_{k^*} R_{ij^*} = \partial_{j^*} R_{ik^*}. \tag{2.18}$$

The contracted Bianchi identities are automatically satisfied in view of the relation

$$R_{ij^*} = -\partial_i \partial_{j^*} \{\ln \det(g_{kl^*})\}. \tag{2.19}$$

Acting with covariant derivatives on the Kähler potential yields

$$\begin{aligned}
\delta_i \delta_{j^*} K &= \partial_i \partial_{j^*} K = g_{ij^*}, & (2.20)\\
\delta^i \delta_j K &= g^{ik^*} g_{jk^*} = \delta^i{}_j, & (2.21)\\
\delta^{i^*} \delta_{j^*} K &= \delta^{i^*}{}_{j^*}. & (2.22)
\end{aligned}$$

The noncommutativity of the covariant derivatives is described by the Riemann tensor, in general:

$$[\delta_a, \delta_b] X^c = -R^c{}_{dab}\, X^d. \tag{2.23}$$

For the particular case of a Kähler manifold

$$\begin{aligned}
[\delta_i, \delta_{j^*}] X^k &= -R^k{}_{mij^*}\, X^m, & (2.24)\\
[\delta_i,\, \delta_j] X^k &= 0, & (2.25)\\
[\delta_i, \delta_{j^*}] X_k &= +R^m{}_{kij^*}\, X_m, & (2.26)\\
[\delta_i,\, \delta_k] X_k &= 0, & (2.27)\\
& \quad \textit{etc.}
\end{aligned}$$

On a Kähler manifold it is possible to define two distinct Laplacians

$$\begin{aligned}
\Delta &= \delta^i \delta_i, & (2.28)\\
\overline{\Delta} &= (\delta^i \delta_i)^* = \delta^{i^*} \delta_{i^*} = \delta_i \delta^i \not\equiv \Delta. & (2.29)
\end{aligned}$$

Acting on scalars $\Delta = \overline{\Delta}$; acting on vectors however they differ by a term proportional to the Ricci tensor

$$\begin{aligned}
(\Delta - \overline{\Delta}) X^k &= -g^{ij^*}\, [\delta_i, \delta_{j^*}]\, X^k & (2.30)\\
&= +g^{ij^*}\, R^k{}_{lij^*}\, X^l & (2.31)\\
&= R^k{}_l\, X^l. & (2.32)
\end{aligned}$$

The lack of commutativity of the covariant derivatives will prove useful when calculating the scalar mass matrix.

# Chapter 3

# The Lagrangian

The superspace form of the action for N=1 supergravity coupled to gauged chiral matter is given in Cremmer *et al.* [2].

$$\int \mathcal{L}\, d^4x \;=\; \int d^4x\, d^4\theta\, E \Big( -3\ \exp\left[-\tfrac{1}{3}K\left(\phi, \overline{\phi}\, e^{2V}\right)\right] + \mathrm{Re}\left[\frac{1}{R}\, W(\phi)\right] + \mathrm{Re}\left[\frac{1}{R}\, f_{\alpha\beta}(\phi)\, W_a^{\alpha}(V)\, \epsilon^{ab}\, W_b^{\beta}(V)\right]\Big). \tag{3.1}$$

Here:

- $E$ is the superspace determinant.
- $R$ is the chiral scalar curvature superfield.
- $\phi$ is the superfield describing chiral matter.
- $V$ is the superfield describing the gauge multiplet.
- $W_a^{\alpha}(V)$ is the field strength superfield, a function of V.
- $K$ is the Kähler potential.
- $W(\phi)$ is the superpotential, a function of $\phi$ only.
- $f_{\alpha\beta}(\phi)$ describes the noncanonical kinetic energy terms of the gauge bosons; it is a function of $\phi$ only.

Both $W$ and $f_{\alpha\beta}$ are analytic, since they are functions of $\phi$ only. However they need not be entire— isolated singularities and/or branch cuts are acceptable. There will be more discussion of this point later. The superpotential $W$ may be thought of as a scalar, though at a more technical level Witten and Bagger [1] have shown the superpotential should actually be interpreted as an analytic section of some holomorphic line bundle whose base space is the Kähler manifold of scalar fields. The gauge metric $f_{\alpha\beta}(\phi)$ is an analytic function transforming as a symmetric tensor in the adjoint representation of the gauge group.

Note that the chiral matter has been gauged by making the substitution

$$K\left(\phi, \overline{\phi}\right) \mapsto K\left(\phi, \overline{\phi}\, e^{2V}\right). \tag{3.2}$$

Thus the Lagrangian of Cremmer *et al.* [2] presupposes that the gauge group acts on the chiral matter fields according to a linear representation. Bagger [1] has extended this analysis to the gauging of symmetries represented nonlinearly on the Kähler manifold, but we shall not take up this particular possibility. Following Cremmer *et al.* then, we demand that the gauge group acts linearly on the Kähler manifold $\mathcal{M}$. This condition is obviously not maintained under arbitrary coordinate reparameterizations of the Kähler manifold $\mathcal{M}$. Though there are many coordinate systems available on the Kähler manifold, we shall only be interested in using a restricted set of coordinate systems, namely those coordinate systems in which the action of the gauge group is linear.

$$\phi^i \mapsto \phi^i + \delta_G \phi^i; \qquad \delta_G \phi^i = i(\xi^\alpha \, [t_\alpha{}^i{}_j] \, \phi^j) \tag{3.3}$$

$$\phi^{i^*} \mapsto \phi^{i^*} + \delta_G \phi^{i^*}; \qquad \delta_G \phi^{i^*} = -i(\xi^\alpha \, [t_\alpha{}^{i^*}{}_{j^*}] \, \phi^{j^*}) \tag{3.4}$$

Here the $\xi^\alpha$ are a set of real parameters while the $[t_\alpha{}^i{}_j]$ are a (in general reducible) representation of the gauge symmetry Lie algebra.

$$[\,\text{Note}: \quad t_\alpha{}^{i^*}{}_{j^*} = (t_\alpha{}^i{}_j)^*]. \tag{3.5}$$

The invariance of the action under the action of the gauge symmetry thus forces us to take

- 1) K the Kähler potential is an <u>invariant</u>.
- 2) W the superpotential is an <u>invariant</u>.

- 3) $f_{\alpha\beta}$ the gauge metric is covariant.

In terms of explicit coordinates then

$$\delta_G K = 0 = \frac{\partial K}{\partial \phi^i}\ \delta_G \phi^i + \frac{\partial K}{\partial \phi^{i^*}}\ \delta_G \phi^{i^*} \tag{3.6}$$

so

$$\left(\frac{\partial K}{\partial \phi^i}\ [t_\alpha{}^i{}_j]\ \phi^j\right) = \left(\frac{\partial K}{\partial \phi^{i^*}}\ [t_\alpha{}^{i^*}{}_{j^*}]\ \phi^{j^*}\right) = \left(\frac{\partial K}{\partial \phi^i}\ [t_\alpha{}^i{}_j]\ \phi^j\right)^* . \tag{3.7}$$

While for the superpotential we see

$$\delta_G W = 0 = \frac{\partial W}{\partial \phi^i}\ \delta_G \phi^i \tag{3.8}$$

so that

$$\frac{\partial W}{\partial \phi^i}\ [t_\alpha{}^i{}_j]\ \phi^j = 0. \tag{3.9}$$

To discuss the behaviour of $f_{\alpha\beta}$ first recall that the $t_\alpha$ constitute a (possibly reducible) representation of the gauge group

$$[t_\alpha,\ t_\beta] = i c_{\alpha\beta}{}^\gamma\ \ t_\gamma. \tag{3.10}$$

Then since $f_{\alpha\beta}$ is covariant in the symmetric adjoint representation

$$\delta_G f_{\alpha\beta} = \frac{\partial f_{\alpha\beta}}{\partial \phi^i}\ \delta_G \phi^i, \tag{3.11}$$

$$\frac{\partial f_{\alpha\beta}}{\partial \phi^i}\ [t_\gamma{}^i{}_j]\ \phi^j = c^\sigma_{\gamma\alpha}\ f_{\sigma\beta} + c^\sigma_{\gamma\beta}\ f_{\alpha\sigma}. \tag{3.12}$$

Note that my generators are normalized in a nonstandard way so that they contain a factor of the gauge coupling constant.

The geometrical interpretation of the gauge metric $f_{\alpha\beta}(\phi)$ is not particularly obvious. Let us interpret the gauge fields ($A^\alpha, \lambda^\alpha$, *etc.*) as lying in some vector bundle over the Kähler manifold $\mathcal{M}$. While $f_{\alpha\beta}$ itself is complex analytic the real part $f^R{}_{\alpha\beta} = \frac{1}{2}(f_{\alpha\beta} + \overline{f_{\alpha\beta}})$ is real and symmetric. If we assume that $f^R{}_{\alpha\beta}$ is positive definite over all of $\mathcal{M}$ then we can interpret $f^R{}_{\alpha\beta}(\phi)$ as a metric in the vector space fibre over $\phi$. This now raises the question as to the appropriate definition for the covariant derivative acting on $f_{\alpha\beta}$ . We shall define the covariant derivative acting on $f_{\alpha\beta}$ to be just the ordinary derivative

$$\delta_i f_{\alpha\beta} = \partial_i f_{\alpha\beta}, \tag{3.13}$$

$$\delta_{i^*} f_{\alpha\beta} = 0. \tag{3.14}$$

At first glance, this definition looks highly noncovariant. We shall now give it a proper geometrical interpretation.

If we make no assumptions concerning the affine connexion in the vector bundle we may write

$$\delta_i(t_\alpha{}^j{}_k) = \partial_i(t_\alpha{}^j{}_k) - \Gamma^\beta{}_{\alpha i}\,[t_\beta{}^j{}_k] + \Gamma^j{}_{mi}\,[t_\alpha{}^m{}_k] - \Gamma^m{}_{ki}\,[t_\alpha{}^j{}_m], \tag{3.15}$$

$$\delta_{i^*}(t_\alpha{}^j{}_k) = \partial_{i^*}(t_\alpha{}^j{}_k) - \Gamma^\beta_{\alpha i^*}\,[t_\beta{}^j{}_k]. \tag{3.16}$$

Recall that the mixed components $\Gamma^i{}_{jk^*}$ are zero. Now we have already argued that we should choose the coordinate system on M so that the gauge group is realized linearly. In particular this implies that the representation matrices are constants,

$$\partial_i[t_\alpha{}^j{}_k] = 0 = \partial_{i^*}[t_\alpha{}^j{}_k]. \tag{3.17}$$

Then

$$\delta_i(t_\alpha{}^j{}_k) = -\Gamma^\beta{}_{\alpha i}\,[t_\beta{}^j{}_k] + \Gamma^j{}_{mi}\,[t_\alpha{}^m{}_k] + \Gamma^m{}_{ki}\,[t_\alpha{}^j{}_m], \tag{3.18}$$

$$\delta_{i^*}(t_\alpha{}^j{}_k) = -\Gamma^\beta{}_{\alpha i^*}\,[t_\beta{}^j{}_k]. \tag{3.19}$$

It is now natural to demand the covariant constraint

$$\delta_i(t_\alpha{}^j{}_k) = 0 = \delta_{i^*}(t_\alpha{}^j{}_k). \tag{3.20}$$

The covariant constancy of the generators then implies that in the special class of coordinate systems where the gauge group is realized linearly

$$\Gamma^\beta{}_{\alpha i} = 0, \tag{3.21}$$

$$\Gamma^j{}_{mi}\,[t_\alpha{}^m{}_k] - \Gamma^m{}_{ki}\,[t_\alpha{}^j{}_m] = 0. \tag{3.22}$$

The first of these relationships implies that in this special class of coordinate systems

$$\delta_i f_{\alpha\beta} = \partial_i f_{\alpha\beta}, \tag{3.23}$$

$$\delta_{i^*} f_{\alpha\beta} = 0. \tag{3.24}$$

This now provides a geometrical interpretation of the Kähler covariant derivative acting on adjoint indices in terms of the covariant constancy of the generators.

To check the consistency of this interpretation we should show that in the special linear class of coordinate systems

$$\Gamma^j{}_{mi}\,[t_\alpha{}^m{}_k] = \Gamma^m{}_{ki}\,[t_\alpha{}^j{}_m]. \tag{3.25}$$

This is in fact easily seen as follows. The vector fields that generate infinitesimal gauge transformations are

$$V_\alpha{}^i = i([t_\alpha{}^i{}_j]\ \ \phi^j). \tag{3.26}$$

The related one-forms

$$V_{\alpha i^*} = i(g_{i^*k}\,[t_\alpha{}^k{}_j]\,\phi^j) \tag{3.27}$$

are by hypothesis Killing one-forms of the metric $g_{ij^*}$ [1]. This implies

$$g_{i^*k}\,[t_\alpha{}^k{}_j] = g_{jk^*}\,[t_\alpha{}^{k^*}{}_{i^*}]. \tag{3.28}$$

Differentiate with respect to $\phi^l$

$$\partial_l g_{i^*k}\,[t_\alpha{}^k{}_j] = \partial_l g_{jk^*}\,[t_\alpha{}^{k^*}{}_{i^*}]. \tag{3.29}$$

$$\begin{aligned}
\Gamma^i{}_{kl}\,[t_\alpha{}^k{}_j] &= \Gamma_{k^*lj}\,[t_\alpha{}^{k^*}{}_{i^*}]\,g^{ii^*} && (3.30)\\
&= \Gamma_{k^*lj}\,g^{k^*k}\,[t_\alpha{}^i{}_k] && (3.31)\\
&= \Gamma^k{}_{jl}\,[t_\alpha{}^i{}_k] \quad \text{as required.} && (3.32)
\end{aligned}$$

Our proposed definition of the covariant derivative of the gauge metric is thus consistent. By hypothesis $f^R{}_{\alpha\beta}$ is positive definite and symmetric. Therefore the inverse $(f_R{}^{-1})^{\alpha\beta}$ exists and is well defined over the whole of the Kähler manifold $\mathcal{M}$. $f^R$ and $f_R{}^{-1}$ will be used to raise and lower adjoint indices. Note however that the action of raising and lowering adjoint indices does not commute with the action of the covariant derivative. Finally, observe

$$\delta_i(f_R{}^{-1})^{\alpha\beta} = \partial_i(f_R{}^{-1})^{\alpha\beta} = -(f_R{}^{-1})^{\alpha\sigma}(\delta_i f^R_{\sigma\rho})(f_R{}^{-1})^{\rho\beta}, \tag{3.33}$$

$$\delta_i f^R_{\alpha\beta} = \partial_i f^R_{\alpha\beta} = \tfrac{1}{2}\partial_i f_{\alpha\beta} = \tfrac{1}{2}\delta_i f_{\alpha\beta}. \tag{3.34}$$

We are now almost ready to write down the component Lagrangian for N=1 supergravity coupled to gauged chiral matter.

Let us define

$$G = K + \ln W + \ln \overline{W}, \tag{3.35}$$

and further

$$D_\alpha = \delta_i G \, [t_\alpha{}^i{}_j] \, \phi^j = \delta_i K \, [t_\alpha{}^i{}_j] \, \phi^j = \overline{D_\alpha}. \tag{3.36}$$

Define a differential operator $d$ by

$$d = (\delta^i G) \, \delta_i = (g^{ij^*} \, \delta_{j^*} G) \, \delta_i, \tag{3.37}$$

so that

$$(df)_{\alpha\beta} = \delta^i G \;\; \delta_i f_{\alpha\beta}. \tag{3.38}$$

Following Cremmer *et al.* [2], we split the Lagrangian into

$$\mathcal{L} = \mathcal{L}_B + \mathcal{L}_{F,K} + \mathcal{L}_{F,M}. \tag{3.39}$$

The individual pieces may now be transcribed as follows:

$$\begin{aligned} e^{-1} \, \mathcal{L}_B \;\; = \;\; & -g_{ij^*} \, D_\mu \phi^i \, D^\mu \phi^{j^*} \\ & -\tfrac{1}{4} f^R{}_{\alpha\beta} \, F^\alpha{}_{\mu\nu} \, F^{\beta\,\mu\nu} \\ & -\tfrac{i}{4} f^I{}_{\alpha\beta} \, F^\alpha{}_{\mu\nu} \, \tilde{F}^{\beta\;\mu\nu} \\ & -\tfrac{1}{2} R \\ & -e^G \, (\delta_i G \, \delta^i G - 3) - \tfrac{1}{2} \, D_\alpha \, (f_R{}^{-1})^{\alpha\beta} \, D_\beta. \end{aligned} \tag{3.40}$$

$$\begin{aligned} e^{-1} \, \mathcal{L}_{F,K} \;\; = \;\; & -\tfrac{1}{2} f^R{}_{\alpha\beta} \, \overline{\lambda}^\alpha (\gamma \cdot D) \lambda^\beta \\ & -\{ g_{ij^*} \overline{\chi}_L{}^i (\gamma \cdot D) \chi_R{}^{j^*} + \text{h.c.} \} \\ & -\tfrac{1}{4} e^{-1} \epsilon^{\mu\nu\rho\sigma} \{ (\overline{\psi}_\mu \gamma_5 \gamma_\nu D_\rho \psi_\sigma) + \text{h.c.} \} \\ & -\tfrac{i}{4} e^{-1} f^I{}_{\alpha\beta} D_\mu (e \, \overline{\lambda}^\alpha \gamma_5 \gamma^\mu \, \lambda^\beta) \\ & +\tfrac{1}{4} f^R{}_{\alpha\beta} \{ (\overline{\lambda}_L{}^\alpha \gamma^\mu \lambda_R{}^\beta)(\delta_i G \, D_\mu \phi^i) + \text{h.c.} \} \\ & +\tfrac{1}{4} f^R{}_{\alpha\beta} \{ (\overline{\lambda}^\alpha \gamma^\mu (\sigma \cdot F)^\beta \psi_\mu) + \text{h.c.} \} \\ & -\tfrac{1}{2} \{ (\overline{\chi}_L{}^i \, \delta_i f_{\alpha\beta} \, (\sigma \cdot F)^\alpha \lambda_L{}^\beta) + \text{h.c.} \} \\ & -\tfrac{1}{8} e^{-1} \epsilon^{\mu\nu\rho\sigma} (\overline{\psi}_\mu \gamma_\nu \psi_\rho)(\delta_i G \, D_\sigma \phi^i + \delta_{i^*} G \, D_\sigma \phi^{i^*}) \\ & +\{ g_{ij^*} (\overline{\psi}_{L\mu} ((\gamma \cdot D) \phi^{j^*}) \gamma^\mu \chi_L{}^i) + \text{h.c.} \} \\ & +\{ (\overline{\chi}_L{}^i ((\gamma \cdot D) \phi^j) \chi_R{}^{k^*})(\partial_i \partial_j \partial_{k^*} G - \tfrac{1}{2} g_{ik^*} \delta_j G) + \text{h.c.} \}. \end{aligned} \tag{3.41}$$

$$
\begin{aligned}
e^{-1}\,\mathcal{L}_{F,M} \;\; = \;\; & +e^{G}\,\overline{\psi}_{\mu}\sigma^{\mu\nu}\psi_{\nu} \\
& +\tfrac{1}{4}e^{G/2}\{\overline{df}_{\alpha\beta}\overline{\lambda}_{R}{}^{\alpha}\lambda_{R}{}^{\beta}+df_{\alpha\beta}\overline{\lambda}_{L}{}^{\alpha}\lambda_{L}{}^{\beta}\} \\
& -e^{G/2}\{(\delta_{i}\delta_{j}G+\delta_{i}G\;\delta_{j}G)(\overline{\chi}_{L}{}^{i}\chi_{L}{}^{j})\} \\
& -e^{G/2}\{(\delta_{i^*}\delta_{j^*}G+\delta_{i^*}G\delta_{j^*}G)(\overline{\chi}_{R}{}^{i^*}\chi_{R}{}^{j^*})\} \\
& +\tfrac{1}{2}D_{\alpha}\{(i\overline{\psi}_{L}\cdot\gamma\lambda_{R}{}^{\alpha})+\text{h.c.}\} \\
& +e^{G/2}\{(\overline{\psi}_{R}\cdot\gamma\;[\delta_{i}G]\;\chi_{L}{}^{i})+\text{h.c.}\} \\
& -\{[2i\;(\delta_{i^*}D_{\alpha})\;(\overline{\lambda}_{R}{}^{\alpha}\chi_{R}{}^{i^*})]+\text{h.c.}\} \\
& -\tfrac{1}{2}D^{\alpha}\{(i\;(\delta_{j}f_{\alpha\beta})\;(\overline{\chi}_{L}{}^{j}\lambda_{L}{}^{\beta}))+\text{h.c.}\} \\
& -\tfrac{1}{16}\{(g^{ij^*}[\delta_{i}f_{\alpha\beta}]\;[\delta_{j^*}f_{\gamma\delta}])(\overline{\lambda}_{L}{}^{\alpha}\lambda_{L}{}^{\beta})(\overline{\lambda}_{R}{}^{\gamma}\lambda_{R}{}^{\delta})\} \\
& +\tfrac{3}{16}\{(f^{R}{}_{\alpha\beta}[\overline{\lambda}_{L}{}^{\alpha}\gamma_{m}\lambda_{R}{}^{\beta}])^{2}\} \\
& +\tfrac{1}{8}f^{R}{}_{\alpha\beta}\{(\overline{\lambda}^{\alpha}\gamma^{\mu}\sigma^{\rho\sigma}\psi_{\mu})(\overline{\psi}_{\rho}\gamma_{\sigma}\lambda^{\beta})+\text{h.c.}\} \\
& +\tfrac{1}{2}\{(\delta_{i}f_{\alpha\beta}\;(\overline{\chi}_{L}{}^{i}\sigma^{\mu\nu}\lambda_{L}{}^{\alpha})\;(\overline{\psi}_{L\nu}\gamma_{\mu}\lambda_{R}{}^{\beta}))+\text{h.c.}\} \\
& +\tfrac{1}{8}\{(\delta_{i}f_{\alpha\beta}\;(\overline{\psi}_{R}\cdot\gamma\;\chi_{L}{}^{i})\;(\overline{\lambda}_{L}{}^{\alpha}\lambda_{L}{}^{\beta}))+\text{h.c.}\} \\
& -\tfrac{1}{8}\{[(g_{ij^*}\overline{\chi}_{R}{}^{j^*}\gamma_{d}\;\chi_{L}{}^{i})(\epsilon^{abcd}[\overline{\psi}_{a}\gamma_{b}\psi_{c}]-[\overline{\psi}^{a}\gamma_{5}\gamma^{d}\psi_{a}])]+\text{h.c.}\} \\
& +\tfrac{1}{8}\{[(g_{ij^*}\overline{\chi}_{L}{}^{i}\gamma^{\mu}\chi_{R}{}^{j^*})(f^{R}{}_{\alpha\beta}\overline{\lambda}_{R}{}^{\alpha}\gamma_{\mu}\lambda_{L}{}^{\gamma})]+\text{h.c.}\} \\
& +\tfrac{1}{16}\{[(\delta_{i}f_{\beta\sigma}(f_{R}{}^{-1})^{\sigma\rho}\;\delta_{j^*}\overline{f}_{\rho\alpha})(\overline{\chi}_{L}{}^{i}\gamma_{\mu}\;\chi_{R}{}^{j^*})(\overline{\lambda}_{R}{}^{\alpha}\gamma^{\mu}\;\lambda_{L}{}^{\beta})]+\text{h.c.}\} \\
& +\tfrac{1}{4}\{[(\overline{\chi}_{L}{}^{i}\chi_{L}{}^{j})(\overline{\lambda}_{L}{}^{\alpha}\lambda_{L}{}^{\beta})(\delta_{i}\delta_{j}f_{\alpha\beta}-\tfrac{1}{4}\delta_{i}f_{\alpha\sigma}(f_{R}{}^{-1})^{\sigma\rho}\;\delta_{j}f_{\rho\beta})]+\text{h.c.}\} \\
& -\tfrac{1}{16}\{[(\overline{\chi}_{L}{}^{i}\sigma^{\mu\nu}\;\chi_{L}{}^{j})(\overline{\lambda}_{L}{}^{\alpha}\sigma_{\mu\nu}\lambda_{L}{}^{\beta})(\delta_{i}f_{\alpha\sigma}(f_{R}{}^{-1})^{\sigma\rho}\;\delta_{j}f_{\rho\beta})]+\text{h.c.}\} \\
& +\tfrac{1}{2}\{[(R_{ij^*kl^*}-\tfrac{1}{2}g_{ij^*}g_{kl^*})(\overline{\chi}_{L}{}^{i}\chi_{L}{}^{k})(\overline{\chi}_{R}{}^{j^*}\;\chi_{R}{}^{l^*})]+\text{h.c.}\}
\end{aligned}
\qquad (3.42)
$$

This component Lagrangian should be compared to the ones exhibited by Bagger [1], by Witten and Bagger [1], and to the Lagrangian exhibited by Cremmer *et al.* [2]. Note that the benefits of the covariant notation at this stage appear to be modest. The benefits at this stage amount to the recognition of the presence of the Riemann tensor in the quartic spin 1/2 term, the mild simplification of the quadratic spin 1/2 term when written using covariant derivatives, and the suppression of many explicit occurrences of the metric. It is true that the covariant notation has not lead to any great simplification in the component Lagrangian itself. However when we proceed to the calculation of mass matrices the covariant notation will lead to immense simplifications.

For the expert in the field I shall now give a brief critique contrasting the covariant notation used here with the notation of Cremmer *et al.* [2], and that of Bagger [1].

(1) In the notation of Cremmer *et al.* [2] the Kähler metric $G''{}_i{}^j$ is negative definite. To conform to standard usage I replace $G$ by $-G$ wherever it occurs in the expressions of Cremmer *et al.* This is trivial, nevertheless it does improve the readability of subsequent calculations.

(2) The index usage is completely different. Cremmer *et al.* only distinguish between index up and index down. The covariant notation uses four types of index; up without star; down without star; up with star and down with star;

$$X^i; \quad X_i; \quad X^{i^*}; \quad X_{i^*}. \tag{3.43}$$

These index conventions may be related to those of Cremmer *et al.* by inspection of the following table:

| Cremmer *et al.* | Covariant Notation |
|---|---|
| $\phi_j$<br>$\phi^j = (\phi_j)^*$ | $\phi^j$<br>$\phi^{j^*} = (\phi^j)^*$ |
| $X_j$<br>$Y^j$<br>$G''{}_i{}^j X_j$<br>$G''{}_j{}^i Y^j$ | $X^j$<br>$Y^{j^*}$<br>$X_{i^*} = g_{i^*j}\, X^j$<br>$Y_i = g_{ij^*}\, Y^{j^*}$ |
| $G''{}_i{}^j G'''{}_j{}^{kl}$ | $g^{ij^*}\partial_{j^*}\partial_k\partial_l G = g^{ij^*}\partial_k(g_{lj^*}) = \Gamma^i{}_{lk}$ |

As an immediate consequence of this table we see that many explicit occurrences of $G''$ and $G''^{-1}$ may be swept under the rug by being replaced by appropriate index contractions. Occurrences of $G'''$ may often be converted into Christoffel symbols and then combined with partial derivatives to yield covariant derivatives.

(3) Note that the indices occurring on the spin 1/2 partners of the scalar fields are related to their chirality thusly

$$\chi_L{}^i; \quad \overline{\chi}_L{}^i \quad \text{and} \qquad \chi_R{}^{i^*}; \quad \overline{\chi}_R{}^{i^*}. \tag{3.44}$$

Bagger [1] does not choose to use this notation and must instead rely upon explicitly exhibiting $(1+\gamma_5)$ factors in his Lagrangian. This leads to odd looking (but correct) terms involving contractions such as $X^{i^*}Y_i$, *i.e.* contracting on mixed starred and unstarred indices. That the left and right chirality pieces of $\chi$ transform differently follows from the fact that $\chi$ is a Majorana spinor:

$$\chi = C\,\overline{\chi}^T = C\,\gamma^0\,\chi^*, \tag{3.45}$$

therefore

$$\chi_R = C\,\gamma^0\,(\chi_L)^*. \tag{3.46}$$

So if $\chi_L \mapsto U\chi_L$ then $\chi_R \mapsto U^*\chi_R$, and so the left and right chirality pieces of $\chi$ transform according to complex conjugate representations.

This completes our comparison of the various notations and we now turn attention to the computation of various mass matrices.

# Chapter 4

# The Mass Matrices

The tree level mass matrices are obtained by looking at the quadratic pieces of the expansion of the Lagrangian $\mathcal{L}$ about the vacuum. Note that it is not sufficient to look at the quadratic pieces in the potential, it is also necessary to investigate the behaviour of the kinetic energy terms at the vacuum. This arises because the noncanonical form of the kinetic energy terms introduces what may be thought of as a tree level wave function renormalization, which must be eliminated in order to properly normalize the fluctuations and so define the masses.

The scalar potential of N=1 supergravity coupled to gauged chiral matter is given by

$$\begin{aligned} V &= e^G(\delta_i G\ \delta^i G - 3) + \tfrac{1}{2}\ D_\alpha\ D^\alpha && (4.1) \\ &= e^G(\delta_i G\ \delta^i G - 3) + \tfrac{1}{2}\ ({f_R}^{-1})^{\alpha\beta}\ D_\alpha\ D_\beta && (4.2) \end{aligned}$$

The vacuum by definition occurs at a minimum of $V$ (*i.e.* $\delta_i V = 0$). Further the vacuum is assumed to satisfy $V|_{vacuum} = 0$ so that there is no induced cosmological constant at tree level.

Observe that

$$\begin{aligned} \delta_i V &= e^G[(\delta_i\delta_j G + \delta_i G\ \delta_j G)\ \delta^j G - 2\ \delta_i G] + \tfrac{1}{2}\ \delta_i(D_\alpha\ D^\alpha) && (4.3) \\ &= e^G[(\delta_i\delta_j G + \delta_i G\ \delta_j G)\ \delta^j G - 2\ \delta_i G] \\ &\qquad +(\delta_i D_\alpha)\ D^\alpha - \tfrac{1}{2}(\delta_i\ {f^R}_{\alpha\beta})\ D^\alpha D^\beta. && (4.4) \end{aligned}$$

It is sometimes more convenient to separately define

$$V_0 = e^G(\delta_i G\ \delta^i G - 3) \qquad (4.5)$$

so that

$$V = V_0 + \tfrac{1}{2} D_\alpha D^\alpha. \tag{4.6}$$

Note that $V_0$ is the limit of $V$ as the gauge coupling $g$ is set to zero. Again observe that

$$\delta_i V_0 = e^G [(\delta_i \delta_j G + \delta_i G\, \delta_j G)\, \delta^j G - 2\, \delta_i G]. \tag{4.7}$$

This particular combination of terms shall appear many times in the calculations that follow.

Before proceeding with the calculations we introduce the concept of the vielbein. We have two metrics present in the problem, one for matter ($g_{ij^*}$) and one for radiation (${f^R}_{\alpha\beta}$), we define vielbeins $e$ and $h$ by

$$g_{ij^*} = {e_i}^I\, {e_{j^*}}^{J^*}\, \delta_{IJ^*}, \tag{4.8}$$

$${f^R}_{\alpha\beta} = {h_\alpha}^A\, {h_\beta}^B\, \delta_{AB}. \tag{4.9}$$

The vielbeins have inverses in the usual fashion

$${e_J}^i\, {e_i}^I = {\delta_J}^I, \tag{4.10}$$

$$({e_i}^I)^* = {e_{i^*}}^{I^*}, \tag{4.11}$$

$${h_B}^\alpha\, {h_\alpha}^A = {\delta_B}^A. \tag{4.12}$$

We can use the inverse vielbeins to construct a noncoordinate basis for the tangent space to the Kähler manifold by employing

$$\delta_I = {e_I}^i\, \delta_i. \tag{4.13}$$

In such a basis the commutator of $\delta_I$ with $\delta_{J^*}$ picks up extra contributions due to the aholonomicity of the basis. With these preliminaries disposed of, let us turn to the problem of evaluating the mass matrices.

## Spin 2

The graviton remains massless:

$$\boxed{\boxed{m_2 = 0.}} \tag{4.14}$$

## Spin 3/2

The gravitino acquires a mass:

$$\boxed{\boxed{m_{3/2} = e^{G/2}}} \tag{4.15}$$

In obtaining this mass the gravitino absorbed the "would be Goldstino" and so finally has four polarization states. For more details, see the calculation of the spin 1/2 mass matrix.

## Spin 1

The relevant part of the Lagrangian is:

$$e^{-1}\mathcal{L}_1 = -\frac{1}{4} f^R{}_{\alpha\beta}\, F^\alpha{}_{\mu\nu}\, F^{\beta\,\mu\nu} - g_{ij^*}\, (D_\mu \phi^i)\, (D^\mu \phi^{j^*}), \tag{4.16}$$

where

$$D_\mu \phi^i = (\partial_\mu \delta^i{}_j - i[t_\alpha{}^i{}_j] A^\alpha{}_\mu)\phi^j. \tag{4.17}$$

Suppose the vacuum occurs at the point $\phi_0^j$. Then expanding around the minimum and throwing away fluctuations in $\phi^j$

$$e^{-1}\mathcal{L}_1 = -\frac{1}{4} f^R{}_{\alpha\beta} F^\alpha{}_{\mu\nu} F^{\beta\,\mu\nu} - g_{ij^*}(-i\,[t_\alpha{}^i{}_j]\, A^\alpha{}_\mu\, \phi_0^j)\,(+i\,[t_\beta{}^{j^*}{}_{k^*}]\, A^{\beta\,\mu}\, \phi_0^{k^*}), \tag{4.18}$$

$$e^{-1}\mathcal{L}_1 = -\frac{1}{4} f^R{}_{\alpha\beta} F^\alpha{}_{\mu\nu} F^{\beta\,\mu\nu} - g_{ij^*}(\phi_0^{k^*}\, [t_\beta{}^{j^*}{}_{k^*}]\, g_{ij^*}[t_\alpha{}^i{}_j]\, \phi_0^j)\, A^\alpha{}_\mu\, A^{\beta\,\mu}. \tag{4.19}$$

Now observe

$$\begin{aligned} \delta_i D_\alpha &= \delta_i \left(\delta_{j^*} G\, [t_\alpha{}^{j^*}{}_{k^*}]\, \phi^{k^*}\right) & (4.20)\\ &= (\delta_i \delta_{j^*} G)\, [t_\alpha{}^{j^*}{}_{k^*}]\, \phi^{k^*} & (4.21)\\ &= g_{ij^*}\, [t_\alpha{}^{j^*}{}_{k^*}]\, \phi^{k^*}. & (4.22) \end{aligned}$$

This allows us to write

$$e^{-1}\mathcal{L}_1 = -\frac{1}{4}\, f^R{}_{\alpha\beta}\, F^\alpha{}_{\mu\nu}\, F^{\beta\,\mu\nu} - (\delta_i D_\beta\, \delta^i D_\alpha)\, A^\alpha{}_\mu\, A^{\beta\mu}. \tag{4.23}$$

We define properly normalized gauge boson fields by

$$A^A = h_\alpha{}^A\, A^\alpha. \tag{4.24}$$

Then

$$e^{-1}\mathcal{L}_1 = -\frac{1}{4}\, F^A{}_{\mu\nu}\, F^{A\,\mu\nu} - [h_A{}^\alpha\, h_B{}^\beta\, (\delta_i D_\beta\, \delta^i D_\alpha)]\, A^A{}_\mu\, A^{B\mu}. \tag{4.25}$$

The $(mass)^2$ matrix is now just read off as

$$\boxed{\begin{aligned} (m_1)^2{}_{AB} &= 2\, h_A{}^\alpha\, h_B{}^\beta\, (\delta_i D_\alpha\, \delta^i D_\beta), \\ \mathrm{tr}\,(m_1)^2 &= 2\, (f_R{}^{-1})^{\alpha\beta}\, (\delta_i D_\alpha\, \delta^i D_\beta). \end{aligned}} \tag{4.26}$$

## Spin 1/2

Isolating the spin 1/2 mass matrix requires a little subtlety. The quadratic part of the fermion Lagrangian is:

$$
\begin{aligned}
e^{-1}(\mathcal{L}_{1/2}+\mathcal{L}_{3/2}) \;=\; & -\tfrac{1}{2} f^{R}{}_{\alpha\beta}\, \overline{\lambda}^{\alpha}(\gamma\cdot D)\lambda^{\beta} \\
& -\{g_{ij^*}\overline{\chi}_L{}^{i}(\gamma\cdot D)\chi_R{}^{j^*}+\text{h.c.}\} \\
& -\tfrac{1}{4}e^{-1}\epsilon^{\mu\nu\rho\sigma}\{(\overline{\psi}_\mu\gamma_5\gamma_\nu D_\rho\psi_\sigma)+\text{h.c.}\} \\
& +e^{G/2}\,\overline{\psi}_\mu\sigma^{\mu\nu}\psi_\nu \\
& +\tfrac{1}{4}e^{G/2}\{\overline{df}_{\alpha\beta}\overline{\lambda}_R{}^{\alpha}\lambda_R{}^{\beta}+df_{\alpha\beta}\overline{\lambda}_L{}^{\alpha}\lambda_L{}^{\beta}\} \\
& -e^{G/2}\{(\delta_i\delta_j G+\delta_i G\,\delta_j G)(\overline{\chi}_L{}^{i}\chi_L{}^{j})\} \\
& -e^{G/2}\{(\delta_{i^*}\delta_{j^*}G+\delta_{i^*}G\delta_{j^*}G)(\overline{\chi}_R{}^{i^*}\chi_R{}^{j^*})\} \\
& +\tfrac{1}{2}D_\alpha\{(i\overline{\psi}_L\cdot\gamma\lambda_R{}^{\alpha})+\text{h.c.}\} \\
& +e^{G/2}\{(\overline{\psi}_R\cdot\gamma\,[\delta_i G]\,\chi_L{}^{i})+\text{h.c.}\} \\
& -\{[2i\,(\delta_{i^*}D_\alpha)\,(\overline{\lambda}_R{}^{\alpha}\chi_R{}^{i^*})]+\text{h.c.}\} \\
& -\tfrac{1}{2}D^{\alpha}\{(i\,(\delta_j f_{\alpha\beta})\,(\overline{\chi}_L{}^{j}\lambda_L{}^{\beta}))+\text{h.c.}\}.
\end{aligned}
\tag{4.27}
$$

To get canonical kinetic energies for the spin 1/2 fields we must rescale using the vielbeins. Define

$$
\chi_L{}^{J} = e_j{}^{I}\,\chi_L{}^{j}, \tag{4.28}
$$

$$
\lambda^{A} = \frac{1}{\sqrt{2}}\,h_\alpha{}^{A}\,\lambda^{\alpha}. \tag{4.29}
$$

The somewhat peculiar looking factor of $\sqrt{2}$ in the definition of $\lambda^A$ is not an error. It will prove to be an essential part of the algebra.

After rescaling the spin 1/2 fields,

$$
\begin{aligned}
e^{-1}(\mathcal{L}_{1/2}+\mathcal{L}_{3/2}) \;=\; & -\tfrac{1}{2}\,\overline{\lambda}^{A}(\gamma\cdot D)\lambda^{A} \\
& -\{\overline{\chi}_L{}^{I}(\gamma\cdot D)\chi_R{}^{I^*}+\text{h.c.}\} \\
& -\tfrac{1}{4}e^{-1}\epsilon^{\mu\nu\rho\sigma}\{(\overline{\psi}_\mu\gamma_5\gamma_\nu D_\rho\psi_\sigma)+\text{h.c.}\} \\
& +e^{G/2}\,\overline{\psi}_\mu\sigma^{\mu\nu}\psi_\nu \\
& +\tfrac{1}{2}e^{G/2}\{[h_A{}^{\alpha}\,h_B{}^{\beta}\,\overline{df}_{\alpha\beta}]\overline{\lambda}_R{}^{\alpha}\lambda_R{}^{\beta}+[h_A{}^{\alpha}\,h_B{}^{\beta}\,df_{\alpha\beta}]\overline{\lambda}_L{}^{\alpha}\lambda_L{}^{\beta}\} \\
& -e^{G/2}\{e_I{}^{i}\,e_J{}^{j}\,(\delta_i\delta_j G+\delta_i G\,\delta_j G)(\overline{\chi}_L{}^{I}\chi_L{}^{J})\} \\
& -e^{G/2}\{e_{I^*}{}^{i^*}\,e_{J^*}{}^{j^*}\,(\delta_{i^*}\delta_{j^*}G+\delta_{i^*}G\delta_{j^*}G)(\overline{\chi}_R{}^{I^*}\chi_R{}^{J^*})\}
\end{aligned}
\tag{4.30}
$$

$$
\begin{aligned}
&+\frac{1}{\sqrt{2}}\, D_A \,\{(i\overline{\psi}_L \cdot \gamma \lambda_R{}^A) + \text{h.c.}\} \\
&+e^{G/2}\{(\overline{\psi}_R \cdot \gamma\, [\delta_I G]\, \chi_L{}^I) + \text{h.c.}\} \\
&-\{\sqrt{2}[2i\, (\delta_{i^*} D_\alpha)\, h_A{}^\alpha\, e_{I^*}{}^{i^*}\, (\overline{\lambda}_R{}^\alpha \chi_R{}^{I^*})] + \text{h.c.}\} \\
&-\frac{1}{\sqrt{2}}\, D^\alpha\, \{(i\, (\delta_j f_{\alpha\beta})\, e_J{}^j\, h_B{}^\beta\, (\overline{\chi}_L{}^J \lambda_L{}^\beta)) + \text{h.c.}\}.
\end{aligned}
$$

The "would be Goldstino" may be isolated as that linear combination of $\chi$ and $\lambda$ that couples to the gravitino $\psi$. Thus the "would be Goldstino" $\eta$ is given by

$$
\eta_L \propto \left( e^{G/2}\, [\delta_I G]\, \chi_L{}^I - \frac{i}{\sqrt{2}} D_A\, \lambda_L{}^A \right). \tag{4.31}
$$

To properly fix the normalization, consider the vector

$$
\xi = \frac{1}{\sqrt{3}} \begin{pmatrix} \delta_{I^*} G \\ \frac{i}{\sqrt{2}} e^{-G/2}\, D_A \end{pmatrix} \tag{4.32}
$$

$$
\xi^\dagger = \frac{1}{\sqrt{3}} \left( \delta_I G \;\; ; \;\; -\frac{i}{\sqrt{2}} e^{-G/2}\, D_A \right) \tag{4.33}
$$

then

$$
\begin{aligned}
\xi^\dagger \xi &= \tfrac{1}{3} \left( \delta_I G\, \delta_{I^*} G + \tfrac{1}{2}\, e^{-G}\, D_A\, D_A \right) && (4.34) \\
&= \tfrac{1}{3} \left( 3 + e^{-G}\, V \right) && (4.35) \\
&= 1 + \tfrac{1}{3} e^{-G}\, V && (4.36) \\
&= 1 && (4.37)
\end{aligned}
$$

where we have finally used the condition that $V = 0$ at the minimum.

The properly normalized "would be Goldstino" is now

$$
\eta_L = \xi^\dagger \begin{pmatrix} \chi_L{}^I \\ \lambda_L{}^A \end{pmatrix} = \left( e^{G/2}\, [\delta_I G]\, \chi_L{}^I - \frac{i}{\sqrt{2}} D_A\, \lambda_L{}^A \right). \tag{4.38}
$$

The gravitino–Goldstino coupling may be written as

$$
\sqrt{3}\, e^{G/2}\, \left\{ \overline{\psi}_R \cdot \gamma\, \eta_L \right\} + \text{h.c.} \tag{4.39}
$$

The terms quadratic in spin 1/2 fields can be read directly from the Lagrangian. These quadratic terms will not yet yield the spin 1/2 mass matrix because we have not yet eliminated the contribution of the "would be Goldstino". Let us call the terms quadratic in the spin 1/2 mass matrix the pseudo mass matrix $P_{1/2}$. For left handed fields we have

$$P_{1/2} = \left[ \begin{array}{cc} P_{IJ} & P_{IA} \\ P_{BJ} & P_{AB} \end{array} \right], \tag{4.40}$$

$$P_{IJ} = -e^{G/2}\{e_I{}^i\, e_J{}^j\, [(\delta_i\delta_j G + \delta_i G\, \delta_j G)]\}, \tag{4.41}$$

$$P_{IA} = +i\sqrt{2}\, \{h_A{}^\alpha\, e_I{}^i\, [(\delta_i D_\alpha) - \tfrac{1}{2} D^\beta\, (\delta_i f^R{}_{\alpha\beta})]\}, \tag{4.42}$$

$$P_{AB} = \tfrac{1}{2}\, e^{G/2}\, \{h_A{}^\alpha\, h_B{}^\beta\, df_{\alpha\beta}\}. \tag{4.43}$$

Note that the pseudo mass matrix for right handed fields is just $\overline{P_{1/2}}$, the complex conjugate of $P_{1/2}$.

The key to the problem of fermion masses is to realise that the "would be Goldstino" is an "eigenvector" of the pseudo mass matrix, specifically:

$$P_{1/2}\, \xi = (-2e^{G/2})\, \xi^*. \tag{4.44}$$

This seemingly odd eigenvector equation, with one vector complex conjugated and the other not, is a reflection of the fact that $P_{1/2}$ is a symmetric, complex, but not necessarily Hermitian matrix.

Establishing the "eigenvector" equation is unfortunately a matter of brute force

$$P_{1/2}\, \xi = \frac{1}{\sqrt{3}} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}, \tag{4.45}$$

where

$$\begin{aligned} X_1 &= -e^{G/2}\, e_I{}^i\, (\delta_i\delta_j G + \delta_i G\, \delta_j G)\, \delta^i G \\ &\quad +(i\sqrt{2})(i/\sqrt{2})\, e^{-G/2}\, e_I{}^i \left([\delta_i D_\alpha] - \tfrac{1}{2} D^\beta\, [\delta_i f^R{}_{\alpha\beta}]\right) D^\alpha, \end{aligned} \tag{4.46}$$

$$\begin{aligned} X_2 &= (+i\sqrt{2})\, h_A{}^\alpha\, ([\delta_i D_\alpha] - \tfrac{1}{2} D^\beta [\delta_i f^R{}_{\alpha\beta}])\, \delta^i G \\ &\quad +(\tfrac{1}{2})(+i/\sqrt{2})\, h_A{}^\alpha\, df_{\alpha\beta}\, D^\beta. \end{aligned} \tag{4.47}$$

Now

$$([\delta_i D_\alpha] - \tfrac{1}{2} D^\beta\, [\delta_i f^R \alpha\beta])\, D^\alpha = \delta_i(\tfrac{1}{2} D^\alpha D^\beta [\delta_i f^R \alpha\beta]) = \delta_i(\tfrac{1}{2} D^\alpha D_\alpha). \tag{4.48}$$

While

$$[df_{\alpha\beta}]\ D^{\beta} = \delta^i G\ [\delta_i f_{\alpha\beta}]\ D^{\beta} = 2\delta^i G\ [\delta_i {f^R}_{\alpha\beta}]\ D^{\beta}. \tag{4.49}$$

So

$$P_{1/2}\ \xi = \frac{1}{\sqrt{3}} \begin{pmatrix} -e^{-G/2}\ {e_I}^i\ [e^G\ (\delta_i\delta_j G + \delta_i G\ \delta_j G)\ \delta^i G + \delta_i(\tfrac{1}{2} D_\alpha\ D^\alpha)] \\ (+i\sqrt{2})\ \delta^i G\ [\delta_i D_\alpha]\ {h_A}^\alpha \end{pmatrix} \tag{4.50}$$

But

$$\begin{aligned} \delta^i G\ \delta_i D_\alpha &= \delta^i G\ \delta_i(\delta_{j^*} G\ [{t_\alpha}^{j^*}{}_{k^*}]\ \phi^{k^*}) & (4.51)\\ &= \delta^i G\ g_{ij^*}\ [{t_\alpha}^{j^*}{}_{k^*}]\ \phi^{k^*} & (4.52)\\ &= \delta_{j^*} G\ [{t_\alpha}^{j^*}{}_{k^*}]\ \phi^{k^*} & (4.53)\\ &= D_\alpha. & (4.54) \end{aligned}$$

So

$$P_{1/2}\ \xi = \frac{1}{\sqrt{3}} \begin{pmatrix} -e^{-G/2}\ {e_I}^i\ [\delta_i V + 2e^G\ \delta_i G] \\ +\sqrt{2}\ D_A \end{pmatrix}. \tag{4.55}$$

Finally we use the extremum condition $\delta_i V = 0$ and obtain

$$P_{1/2}\ \xi = (-2e^{G/2})\ \xi^*. \tag{4.56}$$

Having established the "eigenvector" equation we define

$$m_{1/2} = P_{1/2} + 2\ e^{G/2}\ \xi^*\ \xi^\dagger. \tag{4.57}$$

Note that, because $\xi^\dagger \xi = 1 = \xi^T \xi^*$, it follows that $m_{1/2}$ is a complex symmetric matrix satisfying

$$m_{1/2}\ \xi = 0 = \xi^T\ m_{1/2}. \tag{4.58}$$

Now $m_{1/2}$ is in fact the physical mass matrix after eliminating the "would be Goldstino". This may be seen by observing

$$\begin{aligned} (\bar\chi, \bar\lambda)\ P_{1/2} \begin{pmatrix} \chi \\ \lambda \end{pmatrix} &= (\bar\chi, \bar\lambda)\ \left[m_{1/2} - 2e^{G/2} \xi^* \xi^\dagger\right] \begin{pmatrix} \chi \\ \lambda \end{pmatrix} & (4.59)\\ &= \left\{ (\bar\chi, \bar\lambda)\ m_{1/2} \begin{pmatrix} \chi \\ \lambda \end{pmatrix} \right\} - 2e^{G/2}\ \bar\eta\ \eta & (4.60)\\ &= \left\{ (\bar\chi, \bar\lambda)\ [I - \xi^*\ \xi^T]\ m_{1/2}\ [I - \xi\ \xi^\dagger] \begin{pmatrix} \chi \\ \lambda \end{pmatrix} \right\} \\ &\qquad -2e^{G/2}\ \bar\eta\ \eta & (4.61)\\ &= (\bar\chi, \bar\lambda)_{perp}\ m_{1/2} \begin{pmatrix} \chi \\ \lambda \end{pmatrix}_{perp} - 2e^{G/2}\ \bar\eta\ \eta, & (4.62) \end{aligned}$$

thus showing that the fields perpendicular to the "would be Goldstino"' do indeed have mass matrix $m_{1/2}$.

Now observe

$$2\, e^{G/2}\, \xi^* \xi^\dagger = \frac{2}{3}\, e^{G/2} \begin{bmatrix} \delta_I G\, \delta_J G & -\frac{i}{\sqrt{2}} e^{-G/2}\, \delta_I G\, D_A \\ -\frac{i}{\sqrt{2}} e^{-G/2}\, D_B\, \delta_J G & -\frac{1}{2} e^{-G}\, D_A\, D_B \end{bmatrix} \tag{4.63}$$

$$= \frac{2}{3} \begin{bmatrix} e^{G/2} \delta_I G\, \delta_J G & -\frac{i}{\sqrt{2}}\, \delta_I G\, D_A \\ -\frac{i}{\sqrt{2}}\, D_B\, \delta_J G & -\frac{1}{2} e^{-G/2}\, D_A\, D_B \end{bmatrix}. \tag{4.64}$$

Thus we may explicitly evaluate the spin 1/2 mass matrix as

$$\begin{aligned} m_{1/2} &= \begin{bmatrix} m_{IJ} & m_{IB} \\ m_{AJ} & m_{AB} \end{bmatrix} \\ m_{IJ} &= -e^G \{ e_I{}^i\, e_J{}^j\, [\delta_i \delta_j G + \tfrac{1}{3} \delta_i G\, \delta_J G] \} \\ m_{IA} &= +i\sqrt{2} \{ h_A{}^\alpha\, e_I{}^i\, [(\delta_i D_\alpha) - \tfrac{1}{2} D^\beta (\delta_i f^R{}_{\alpha\beta}) - \tfrac{1}{3} (\delta_i G) D_\alpha] \} \\ m_{AB} &= \left\{ h_A{}^\alpha\, h_B{}^\beta\, [\tfrac{1}{2} e^{G/2}\, df_{\alpha\beta} - \tfrac{1}{3} e^{-G/2}\, D_\alpha\, D_\beta] \right\} \end{aligned} \tag{4.65}$$

This finally is the full spin 1/2 mass matrix in all its glory.

To calculate the spin 1/2 contribution to the $(mass)^2$ sum rule we need to evaluate $\mathrm{tr}\,(m_{1/2}\, \overline{m_{1/2}})$. This could be calculated by inserting the mass matrix just calculated and tracing. This is the strategy adopted by Cremmer *et al.* [2]. This strategy is however grossly inefficient when noncanonical kinetic terms are kept. A much more tractable strategy is to observe

$$m_{1/2}\, \overline{m_{1/2}} = (P_{1/2} + 2e^{G/2} \xi^* \xi^\dagger)\, (\overline{P_{1/2}} + 2e^{G/2} \xi \xi^T) \tag{4.66}$$

$$= P_{1/2}\, \overline{P_{1/2}} + 2e^{G/2} \left[ \xi^* (-2e^{G/2} \xi^T) + (-2e^{G/2} \xi^*) \xi^T + 2e^{G/2} \xi^* \xi^T \right] \tag{4.67}$$

$$= P_{1/2}\, \overline{P_{1/2}} - 4e^G \xi^* \xi^T. \tag{4.68}$$

Then

$$\mathrm{tr}\,(m_{1/2}\, \overline{m_{1/2}}) = \mathrm{tr}\,(P_{1/2}\, \overline{P_{1/2}}) - 4e^G. \tag{4.69}$$

But now

$$\begin{aligned}
\mathrm{tr}\,(P_{1/2}\,\overline{P_{1/2}}) &= P_{IJ}\,\overline{P}_{I^*J^*} + 2P_{AI}\,\overline{P}_{I^*A} + P_{AB}\,\overline{P}_{AB} \\
&= e^G\,(\delta_i\delta_j G + \delta_i G\,\delta_j G)\,(\delta^i\delta^j G + \delta^i G\,\delta^j G) \\
&\quad 2[2(\delta_i D_\alpha - \tfrac{1}{2}D^\beta\,\delta_i f^R{}_{\alpha\beta})\,(f_R{}^{-1})^{\alpha\gamma}\,(\delta^i D_\gamma - \tfrac{1}{2}D^\delta\,\delta^i f^R{}_{\delta\gamma})] \\
&\quad \tfrac{1}{4}e^G\,[df_{\alpha\beta}\,(f_R{}^{-1})^{\beta\gamma}\,\overline{df}_{\gamma\delta}\,(f_R{}^{-1})^{\delta\alpha}]. \qquad (4.70)
\end{aligned}$$

Now define

$$\mathrm{tr}\,(df_R\,f_R^{-1}\,\overline{d}f_R\,f_R^{-1}) = \tfrac{1}{4}[df_{\alpha\beta}(f_R{}^{-1})^{\beta\gamma}\,\overline{df}_{\gamma\delta}\,(f_R{}^{-1})^{\delta\alpha}]. \qquad (4.71)$$

Then

$$\begin{aligned}
\mathrm{tr}\,(P_{1/2}\,\overline{P_{1/2}}) &= e^G\,(\delta_i\delta_j G + \delta_i G\,\delta_j G)\,(\delta^i\delta^j G + \delta^i G\,\delta^j G) \\
&\quad +4[\delta_i D_\alpha]\,[\delta^i D_\beta]\,(f^R)^{\alpha\beta} \\
&\quad +e^G\ \mathrm{tr}\,(df_R\,f_R^{-1}\,\overline{d}f_R\,f_R^{-1}) \\
&\quad -2(\delta_i D_\alpha\,(f_R^{-1})^{\alpha\gamma}\,[\delta^i f^R{}_{\gamma\delta}]D^\delta) \\
&\quad -2(\delta^i D_\alpha\,(f_R^{-1})^{\alpha\gamma}\,[\delta_i f^R{}_{\gamma\delta}]D^\delta) \\
&\quad +D_\alpha\,[\delta_i f^R{}_{\alpha\beta}]\,(f_R{}^{-1})^{\beta\gamma}[\delta^i f^R{}_{\gamma\delta}]\,D^\delta. \qquad (4.72)
\end{aligned}$$

Recall that $f_{\alpha\beta}$ transforms like a symmetric tensor in the adjoint representation. This means that

$$\delta_G f_{\alpha\beta} = \partial_i f_{\alpha\beta}\,[t_\gamma{}^i{}_j]\,\phi^i\,\xi^\gamma = (\delta_i f_{\alpha\beta})\,(\delta^i D_\gamma)\,\xi^\gamma, \qquad (4.73)$$

$$(\delta_i f_{\alpha\beta})\,(\delta^i D_\gamma) = c_{\gamma\alpha}{}^\sigma f_{\sigma\beta} + c_{\gamma\beta}{}^\sigma f_{\alpha\sigma}. \qquad (4.74)$$

Applying this, we see that

$$\begin{aligned}
(\delta^i D_\alpha\,(f_R^{-1})^{\alpha\gamma}\,[\delta_i f^R{}_{\gamma\delta}]D^\delta) &= \tfrac{1}{2}[(\delta^i D_\alpha)\,(\delta_i f^R{}_{\gamma\delta})\,(f_R^{-1})^{\gamma\alpha}\,D^\delta \qquad (4.75) \\
&= \tfrac{1}{2}[c_{\alpha\gamma}{}^\sigma f_{\sigma\delta} + c_{\alpha\delta}{}^\sigma f_{\sigma\gamma}]\,(f_R^{-1})^{\gamma\alpha}\,D^\delta \\
&= \tfrac{1}{2}c_{\alpha\delta}{}^\sigma\,(f^R{}_{\sigma\gamma} + if^I{}_{\sigma\gamma})\,(f_R^{-1})^{\gamma\alpha}\,D^\delta \\
&= \tfrac{1}{2}c_{\alpha\delta}{}^\alpha\,D^\delta + i[\tfrac{1}{2}c_{\alpha\delta}{}^\sigma\,(f^I{}_{\sigma\gamma})\,(f_R^{-1})^{\gamma\alpha}\,D^\delta],
\end{aligned}$$

where we have already used the antisymmetry of the structure constants in the first two indices. The purely imaginary piece will be cancelled when we add the complex conjugate term. Also $c_{\alpha\beta}{}^\alpha = 0$ since we can always choose the structure constants to be completely antisymmetric.

Now observe that

$$\Delta({f_R}^{-1}) = \delta^i\delta_i(f_R^{-1}) \quad (4.76)$$

$$= +[({f_R}^{-1})\,\delta^i f_R\,({f_R}^{-1})\,\delta_i f_R] - [({f_R}^{-1})\,\delta^i\delta_i f_R\,({f_R}^{-1})] + [({f_R}^{-1})\,\delta_i f_R\,({f_R}^{-1})\,\delta^i f_R], \quad (4.77)$$

and note that $\delta^i\delta_i f_R = \frac{1}{2}\delta^i(\delta_i f) = 0$, since $f$ is chiral. In particular,

$$D^\alpha[\delta_i {f^R}_{\alpha\beta}]\,({f_R}^{-1})^{\beta\gamma}\,[\delta^i {f^R}_{\gamma\delta}]D^\delta = \frac{1}{2}D_\alpha\,[\delta^i\delta_i({f_R}^{-1})^{\alpha\beta}]\,D_\beta. \quad (4.78)$$

The trace now becomes

$$\begin{aligned}\mathrm{tr}\,(m_{1/2}\,\overline{m_{1/2}}) = \; & e^G\,(\delta_i\delta_j G + \delta_i G\,\delta_j G)\,(\delta^i\delta^j G + \delta^i G\,\delta^j G) \\ & +4(\delta_i D_\alpha)\,({f_R}^{-1})^{\alpha\beta}\,(\delta^i D_\beta) \\ & +e^G\;\mathrm{tr}\,(df_R\,{f_R}^{-1}\,\overline{d}f_R\,{f_R}^{-1}) \\ & +\frac{1}{2}D_\alpha\,[\Delta({f_R}^{-1})^{\alpha\beta}]\,D_\beta \\ & -4e^G. \end{aligned} \quad (4.79)$$

Utilizing the results obtained for higher spin we finally obtain

$$\boxed{\begin{aligned}\mathrm{tr}\,(m_{1/2}\,\overline{m_{1/2}}) = \; & (m_{3/2})^2\,(\delta_i\delta_j G + \delta_i G\,\delta_j G)\,(\delta^i\delta^j G + \delta^i G\,\delta^j G) \\ & +2\,\mathrm{tr}\,(m_1)^2 \\ & (m_{3/2})^2\;\mathrm{tr}\,(df_R\,{f_R}^{-1}\,\overline{d}f_R\,{f_R}^{-1}) \\ & +\frac{1}{2}D_\alpha\,[\Delta({f_R}^{-1})^{\alpha\beta}]\,D_\beta \\ & -4(m_{3/2})^2. \end{aligned}} \quad (4.80)$$

## Spin 0

Finally we calculate the mass matrix for the scalar bosons. The relevant part of the Lagrangian is

$$\begin{aligned} e^{-1}\,\mathcal{L}_0 &= -g_{ij^*}\,D_\mu\phi^i\,D^\mu\phi^{j^*} \\ &\quad -e^G\,(\delta_i G\,\delta^i G-3) \\ &\quad -\tfrac{1}{2}\,D_\alpha\,(f_R{}^{-1})^{\alpha\beta}\,D_\beta && (4.81)\\ &= -g_{ij^*}\,D_\mu\phi^i\,D^\mu\phi^{j^*}-V(\phi,\overline{\phi}). && (4.82)\end{aligned}$$

Now suppose that $V$ has a minimum at $\phi_0^i$ and define the fluctuation $\Delta\phi^i = \phi^i-\phi_0^i$. Now by hypothesis $V$ and $\partial V$ are zero at the minimum. So expanding in terms of fluctuations we see

$$\begin{aligned} e^{-1}\,\mathcal{L}_0 &= -[g_{ij^*}(\phi_0)]\,D_\mu(\Delta\phi^i)\,D^\mu(\Delta\phi^{j^*}) \\ &\quad -(\Delta\phi^{i^*}\;;\;\Delta\phi^i)\begin{bmatrix}\partial_{i^*}\partial_j V & \partial_{i^*}\partial_{j^*}V\\ \partial_i\partial_j V & \partial_i\partial_{j^*}V\end{bmatrix}\begin{pmatrix}\Delta\phi^j\\ \Delta\phi^{j^*}\end{pmatrix} \\ &\quad +O(|\Delta\phi|^3). && (4.83)\end{aligned}$$

Properly normalized fluctuations may be defined using the vielbeins

$$\begin{aligned} \phi^I &= e_i{}^I\,\Delta\phi^i, && (4.84)\\ \phi^{I^*} &= e_{i^*}{}^{I^*}\,\Delta\phi^{i^*}. && (4.85)\end{aligned}$$

Then

$$\begin{aligned} e^{-1}\,\mathcal{L}_0 &= -\partial_\mu\phi^I\,\partial^\mu\phi^{I*} \\ &\quad -(\phi^{I^*}\;;\;\phi^I)\begin{bmatrix} e_{I^*}{}^{i^*}\,e_J{}^j\,\partial_{i^*}\partial_j V & e_{I^*}{}^{i^*}\,e_{J^*}{}^{j^*}\,\partial_{i^*}\partial_{j^*}V\\ e_I{}^i\,e_J{}^j\,\partial_i\partial_j V & e_I{}^i\,e_{J^*}{}^{j^*}\,\partial_i\partial_{j^*}V\end{bmatrix}\begin{pmatrix}\phi^J\\ \phi^{J^*}\end{pmatrix} \\ &\quad +O(|\Delta\phi|^3). && (4.86)\end{aligned}$$

The $(mass)^2$ matrix is now seen to be

$$\begin{aligned} (m_0)^2 &= \begin{bmatrix} e_{I^*}{}^{i^*}\,e_J{}^j\,\partial_{i^*}\partial_j V & e_{I^*}{}^{i^*}\,e_{J^*}{}^{j^*}\,\partial_{i^*}\partial_{j^*}V\\ e_I{}^i\,e_J{}^j\,\partial_i\partial_j V & e_I{}^i\,e_{J^*}{}^{j^*}\,\partial_i\partial_{j^*}V\end{bmatrix} && (4.87)\\ &= \begin{bmatrix} e_{I^*}{}^{i^*}\,e_J{}^j\,\delta_{i^*}\delta_j V & e_{I^*}{}^{i^*}\,e_{J^*}{}^{j^*}\,\delta_{i^*}\delta_{j^*}V\\ e_I{}^i\,e_J{}^j\,\delta_i\delta_j V & e_I{}^i\,e_{J^*}{}^{j^*}\,\delta_i\delta_{j^*}V\end{bmatrix} && (4.88)\\ &= \begin{bmatrix}\delta_{I^*}\delta_J V & \delta_{I^*}\delta_{J^*}V\\ \delta_I\delta_J V & \delta_I\delta_{J^*}V\end{bmatrix}, && (4.89)\end{aligned}$$

where we have repeatedly used the fact that $\partial V = \delta V = 0$ at the minimum.

Evaluating the trace of the $(mass)^2$ matrix is relatively easy. Evaluating the $(mass)^2$ matrix itself is relatively straightforward but tedious. Let us split the problem into manageable chunks as follows

$$\delta_{i^*}\delta_j V = \delta_{i^*}\delta_j V_0 + \delta_{i^*}\delta_j(\tfrac{1}{2}D_\alpha D^\alpha), \tag{4.90}$$

$$\delta_i\delta_j V = \delta_i\delta_j V_0 + \delta_i\delta_j(\tfrac{1}{2}D_\alpha D^\alpha). \tag{4.91}$$

The gauge contributions are

$$\begin{aligned}\delta_{i^*}\delta_j(\tfrac{1}{2}D_\alpha(f_R^{-1})^{\alpha\beta}D_\beta) &= (\delta_{i^*}\delta_j D_\alpha)\ D^\alpha \\ &+\tfrac{1}{2}D_\alpha[\delta_{i^*}\delta_j(f_R^{-1})^{\alpha\beta}]D_\beta \\ &+(\delta_{i^*}D_\alpha)\ (f_R^{-1})^{\alpha\beta}\ (\delta_j D_\beta) \\ &+(\delta_{i^*}D_\alpha)\ [\delta_j(f_R^{-1})^{\alpha\beta}]\ D_\beta \\ &+(\delta_j D_\alpha)\ [\delta_{i^*}(f_R^{-1})^{\alpha\beta}]\ D_\beta. \end{aligned} \tag{4.92}$$

$$\begin{aligned}\delta_i\delta_j(\tfrac{1}{2}D_\alpha(f_R^{-1})^{\alpha\beta}D_\beta) &= (\delta_i\delta_j D_\alpha)\ D^\alpha \\ &+\tfrac{1}{2}D_\alpha[\delta_i\delta_j(f_R^{-1})^{\alpha\beta}]D_\beta \\ &+(\delta_i D_\alpha)\ (f_R^{-1})^{\alpha\beta}\ (\delta_j D_\beta) \\ &+(\delta_i D_\alpha)\ [\delta_j(f_R^{-1})^{\alpha\beta}]\ D_\beta \\ &+(\delta_j D_\alpha)\ [\delta_i(f_R^{-1})^{\alpha\beta}]\ D_\beta. \end{aligned} \tag{4.93}$$

In general, nothing particularly enlightening can be said about the gauge contributions to the $(mass)^2$ matrix. One simplification is to note that $\delta_i\delta_j D_\alpha = 0$, since $\delta_i\delta_j D_\alpha = \delta_i(g_{jk^*}[{t_\alpha}^{k^*}{}_{l^*}]\phi^{l^*}) = 0$. In a similar fashion

$$\delta_{i^*}\delta_j D_\alpha = \delta_{i^*}(g_{jk^*}\ [{t_\alpha}^{k^*}{}_{l^*}]\ \phi^{l^*}) \tag{4.94}$$

$$= g_{jk^*}\ [{t_\alpha}^{k^*}{}_{l^*}]\ {\delta^{l^*}}_{i^*} \tag{4.95}$$

$$= g_{jk^*}\ [{t_\alpha}^{k^*}{}_{i^*}] \tag{4.96}$$

$$= [t_{\alpha j i^*}]. \tag{4.97}$$

Now consider the contribution $\delta\delta V$ to the $(mass)^2$ matrix. We observe

$$\delta_{i^*}\delta_j V_0 = \delta_{i^*}[e^G\ \{(\delta_j\delta_k G + \delta_j G\ \delta_k G)\ \delta^k G - 2\ \delta_j G\}] \tag{4.98}$$

$$\begin{aligned} &= e^G\ [\delta_{i^*}G\ \{(\delta_j\delta_k G + \delta_j G\ \delta_k G)\ \delta^k G - 2\ \delta_j G\} \\ &+\{(\delta_{i^*}\delta_j\delta_k G + \delta_{i^*}\delta_j G\ \delta_k G + \delta_j G\ \delta_{i^*}\delta_k G)\ \delta^k G\} \\ &+\{(\delta_j\delta_k G + \delta_j G\ \delta_k G)\ \delta_{i^*}\delta^k G - 2\ \delta_{i^*}\delta_j G]\} \end{aligned} \tag{4.99}$$

Now recall $\delta_i \delta_{j^*} G = g_{ij^*}$. Also observe

$$
\begin{aligned}
\delta_{i^*}\delta_j\delta_k G &= [\delta_{i^*},\delta_j]\delta_k G + \delta_j\delta_{i^*}\delta_k G &(4.100)\\
&= R^m{}_{ki^*j}\ \delta_m G + \delta_j(g_{ki^*}) &(4.101)\\
&= R^m{}_{ki^*j}\ \delta_m G. &(4.102)
\end{aligned}
$$

Combining these results

$$
\begin{aligned}
\delta_{i^*}\delta_j V_0 &= e^G\ \{(\delta_{i^*}\delta^k G + \delta_{i^*}G\ \delta^k G)\ (\delta_k\delta^j G + \delta_k G\ \delta^j G)\\
&\quad -2g_{i^*j} + g_{i^*j}\ (\delta^k G\ \delta_k G) - \delta_{i^*}G\ \delta_j G\\
&\quad +R^m{}_{ki^*j}\ \delta_m G\ \delta^k G\}. &(4.103)
\end{aligned}
$$

Notice in particular that the Riemann tensor contributes to the scalar masses. The final term of interest is now

$$
\begin{aligned}
\delta_i\delta_j V_0 &= \delta_i[e^G\ \{(\delta_j\delta_k G + \delta_j G\ \delta_k G)\ \delta^k G - 2\ \delta_j G\}] &(4.104)\\
&= e^G[\delta_i G\ \{(\delta_j\delta_k G + \delta_j G\ \delta_k G)\ \delta^k G - 2\ \delta_j G\}\\
&\quad +(\delta_i\delta_j\delta_k G + \delta_i\delta_j G\ \delta_k G + \delta_j G\ \delta_i\delta_k G)\ \delta^k G\\
&\quad +[(\delta_j\delta_k G + \delta_j G\ \delta_k G)\ \delta_i\delta^k G - 2\ \delta_i\delta_j G]] &(4.105)
\end{aligned}
$$

Observing that $\delta^i \delta_j G = \delta^i{}_j$, we see that

$$
\begin{aligned}
\delta_i\delta_j V_0 &= e^G\{(\delta_i\delta_j G + \delta_i G\ \delta_j G) + (\delta_j\delta_j G)\ [\delta_k G\ \delta^k G - 2] + (\delta_i\delta_j\delta_k G)\ \delta^k G\\
&\quad +[(\delta_i\delta_k G)\ \delta^k G\ \delta_j G + (\delta_j\delta_k G)\ \delta^k G\ \delta_i G] + \delta_i G\ \delta_j G\ [\delta_k G\ \delta^k G - 2]\}\\
& &(4.106)\\
&= e^G\ \{(\delta_i\delta_j G + \delta_i G\ \delta_j G)\ [\delta_k G\ \delta^k G - 1] + (\delta_i\delta_j\delta_k G)\ \delta^k G\\
&\quad +(\delta_i\delta_k G)\ \delta^k G\ \delta_j G + (\delta_j\delta_k G)\ \delta^k G\ \delta_i G\}. &(4.107)
\end{aligned}
$$

Now define

$$
\begin{aligned}
G_I &= e_I{}^i\ \delta_i G, &(4.108)\\
G_{IJ} &= e_I{}^i\ e_J{}^j\ \delta_i\delta_j G, &(4.109)\\
G_{IJK} &= e_I{}^i\ e_J{}^j\ e_K{}^k\ \delta_i\delta_j\delta_k G. &(4.110)
\end{aligned}
$$

The $(mass)^2$ matrix decomposes to

$$
\begin{aligned}
(m_0)^2 &= \begin{bmatrix} (m_0)^2{}_{I^*J} & (m_0)^2{}_{I^*J^*} \\ (m_0)^2{}_{IJ} & (m_0)^2{}_{IJ^*} \end{bmatrix}. \\
(m_0)^2{}_{I^*J} &= (m_{3/2})^2\{(\delta_{I^*J} + [G_{I^*}{}^K + G_{I^*}\, G^K]\, [G_{KJ} + G_K\, G^J]) \\
&\qquad +\delta_{I^*J}(G_K\, G^K - 3) - G_{I^*}\, G_J \\
&\qquad +R_{I^*JK}{}^L\, G^K\, G_L\} \\
&\quad +t_{\alpha JI^*} D^\alpha + \delta_{I^*} D_\alpha\, ({f_R}^{-1})^{\alpha\beta}\, \delta_J D_\beta \\
&\quad +\tfrac{1}{2} D_\alpha (e^{i^*}_{I^*}\, e^j_J\, [\delta_{i^*}\delta_j ({f_R}^{-1})^{\alpha\beta}]) D_\beta \\
&\quad +\delta_{I^*} D_\alpha\, \delta_J ({f_R}^{-1})^{\alpha\beta}\, D_\beta + \delta_J D_\alpha\, \delta_{I^*} ({f_R}^{-1})^{\alpha\beta}\, D_\beta. \\
(m_0)^2{}_{IJ} &= (m_{3/2})^2\{(\delta_I\delta_J G + G_I\, G_J)\, (G_K\, G^K - 1) + G_{IJK}\, G^K \\
&\qquad +G_{IK}\, G^K\, G_J + G_{JK}\, G^K\, G_I\} \\
&\quad +\delta_I D_\alpha\, ({f_R}^{-1})^{\alpha\beta}\, \delta_J D_\beta \\
&\quad +\tfrac{1}{2} D_\alpha (e^i_I\, e^j_J\, [\delta_i\delta_j ({f_R}^{-1})^{\alpha\beta}]) D_\beta \\
&\quad +\delta_I D_\alpha\, \delta_J ({f_R}^{-1})^{\alpha\beta}\, D_\beta + \delta_J D_\alpha\, \delta_I ({f_R}^{-1})^{\alpha\beta}\, D_\beta.
\end{aligned}
\tag{4.111}
$$

This finally is the full expression for the scalar $(mass)^2$ matrix. In its present form it is too unwieldy to be of any great use. Some simplifying ansätze will be discussed in subsequent chapters.

Fortunately the trace of the $(mass)^2$ matrix is now very easy to evaluate

$$
\begin{aligned}
\mathrm{tr}\,(m_0)^2 &= (m_0)^2{}_{II^*} + (m_0)^2{}_{I^*I} && (4.112) \\
&= \delta^i\delta_i V + \delta_i\delta^i V && (4.113) \\
&= \Delta V + \overline{\Delta} V && (4.114) \\
&= 2\,\Delta V. && (4.115)
\end{aligned}
$$

since $\Delta = \overline{\Delta}$ when acting on scalars. The explicitly calculated formula for $({m_0}^2)_{I^*J}$ now yields

$$
\begin{aligned}
\mathrm{tr}\,(m_0)^2 &= (m_{3/2})^2\{[n + (\delta_i\delta_j G + \delta_i G\, \delta_j G)\, (\delta^i\delta^j G + \delta^i G\, \delta^j G)] \\
&\qquad +n(\delta_i G\, \delta^i G - 3) \\
&\qquad -\delta_i G\, \delta^i G \\
&\qquad +R^i{}_{ik}{}^l\, \delta^k G\, \delta_l G\}
\end{aligned}
$$

$$
\begin{aligned}
&+2\,\mathrm{tr}\,(t_\alpha) D^\alpha \\
&+2\delta_i D_\alpha\ ({f_R}^{-1})^{\alpha\beta}\ \delta^i D_\beta \\
&+D_\alpha[\Delta({f_R}^{-1})^{\alpha\beta}] D_\beta \\
&+2\delta^i D_\alpha\ \delta_i ({f_R}^{-1})^{\alpha\beta}\ D_\beta \\
&+2\delta_i D_\alpha\ \delta^i ({f_R}^{-1})^{\alpha\beta}\ D_\beta.
\end{aligned} \tag{4.116}
$$

The last two terms occurring here are complex conjugates of each other and have previously been shown to cancel against each other (recall the spin 1/2 calculation). We also use the fact that the condition $V = 0$ at the minimum implies

$$
2\ (m_{3/2})^2\ (\delta_i G\ \delta^i G - 3) = -D_\alpha\ D^\alpha. \tag{4.117}
$$

Additionally, recall

$$
\mathrm{tr}\,(m_1)^2 = 2(\delta_i D_\alpha)\ ({f_R}^{-1})^{\alpha\beta}\ (\delta^i D_\beta). \tag{4.118}
$$

So we see

$$
\begin{aligned}
\mathrm{tr}\,(m_0)^2 \quad = \quad & \{n[2\ (m_{3/2})^2 - D_\alpha\ D^\alpha] \\
&+2(m_{3/2})^2\ (\delta_i\delta_j G + \delta_i G\ \delta_j G)\ (\delta^i\delta^j G + \delta^i G\ \delta^j G) \\
&-[6(m_{3/2})^2 - D_\alpha\ D^\alpha] \\
&+2(m_{3/2})^2\ (R_i{}^j\ \delta^i G\ \delta_j G) \\
&+2\,\mathrm{tr}\,(t_\alpha) D^\alpha \\
&+\mathrm{tr}\,(m_1)^2 \\
&+D_\alpha[\Delta({f_R}^{-1})^{\alpha\beta}] D_\beta.
\end{aligned} \tag{4.119}
$$

And finally

$$
\boxed{
\begin{aligned}
\mathrm{tr}\,(m_0)^2 \quad = \quad & (m_{3/2})^2\ (\delta_i\delta_j G + \delta_i G\ \delta_j G)\ (\delta^i\delta^j G + \delta^i G\ \delta^j G) \\
&(n-1)[2\ (m_{3/2})^2 - D_\alpha\ D^\alpha] \\
&-4(m_{3/2})^2 \\
&+2(m_{3/2})^2\ (\delta^i G\ R_i{}^j\ \delta_j G) \\
&+2\,\mathrm{tr}\,(t_\alpha) D^\alpha \\
&+\mathrm{tr}\,(m_1)^2 \\
&+D_\alpha[\Delta({f_R}^{-1})^{\alpha\beta}] D_\beta.
\end{aligned}
} \tag{4.120}
$$

Having now exhaustively and explicitly evaluated the $(mass)^2$ matrices and their traces, the $(mass)^2$ sum rule itself will be trivial.

# Chapter 5

# The Sum Rule

$$\mathrm{Str}\,(m^2) = \mathrm{tr}\,(m_0^2) - 2\,\mathrm{tr}\,(m_{1/2})^2 + 3\,\mathrm{tr}\,(m_1)^2 - 4\,\mathrm{tr}\,(m_{3/2})^2 \quad (5.1)$$

$$\begin{aligned}
= \;& (n-1)[2m_{3/2}^2 - D_\alpha D^\alpha] \quad (5.2)\\
& +2m_{3/2}^2\,(\delta_i\delta_j G + \delta_i G\,\delta_j G)\,(\delta^i\delta^j G + \delta^i G\,\delta^j G)\\
& -4m_{3/2}^2\\
& +2(m_{3/2})^2\,(\delta^i G\,R_i{}^j\,\delta_j G)\\
& +\mathrm{tr}\,(t_\alpha)\,D^\alpha\\
& +\mathrm{tr}\,(m_1)^2\\
& +D_\alpha[\Delta({f_R}^{-1})^{\alpha\beta}]D_\beta\\
- \;& 2(m_{3/2})^2\,(\delta_i\delta_j G + \delta_i G\,\delta_j G)\,(\delta^i\delta^j G + \delta^i G\,\delta^j G)\\
& -4\,\mathrm{tr}\,(m_1)^2\\
& -2(m_{3/2})^2\,\mathrm{tr}\,(df_R\,{f_R}^{-1}\,\overline{d}f_R\,{f_R}^{-1})\\
& -D_\alpha[\Delta({f_R}^{-1})^{\alpha\beta}]D_\beta\\
& +8(m_{3/2})^2\\
+ \;& 3\,\mathrm{tr}\,(m_1)^2\\
- \;& 4(m_{3/2})^2.
\end{aligned}$$

Collecting terms

$$\boxed{\begin{aligned} \mathrm{Str}\,(m^2) \;=\; & 2(n-1)\,\left[m_{3/2}^2 - \tfrac{1}{2}\,D_\alpha\,D^\alpha\right] \\ & +2\,\mathrm{tr}\,(t_\alpha)\,D^\alpha \\ & +2(m_{3/2})^2\,(\delta^i G\,R_i{}^j\,\delta_j G) \\ & -2(m_{3/2})^2\,\mathrm{tr}\,\left(df_R\,f_R{}^{-1}\,\overline{d}f_R\,f_R{}^{-1}\right). \end{aligned}} \tag{5.3}$$

This should now be compared to the sum rule of Cremmer *et al.* [2]. It is remarkable that the extra contributions due to noncanonical kinetic energies are so simple. When comparing to the results of Cremmer *et al.* [2], recall that $(t_\alpha)_{here} = g(t_\alpha)_{Cremmer}$, and that $(D_\alpha)_{here} = -(D_\alpha)_{Cremmer}$.

Having now exhibited the mass matrices and mass sum rule assuming that $V|_{vacuum} = 0$, we shall turn to the more general question of just how a vanishing cosmological constant may be obtained.

# Chapter 6

# Tuning the Cosmological Constant

The scalar potential for N=1 supergravity coupled to gauged chiral matter is [1,2]

$$V = e^G \, (\delta_i G \, \delta^i G - 3) + \tfrac{1}{2} \, D_\alpha \, D^\alpha. \tag{6.1}$$

We shall define the cosmological constant to be the value of $V$ at its absolute minimum. More carefully we can account for the possibility that the absolute minimum occurs at infinity in field space by defining

$$\Lambda = \inf V. \tag{6.2}$$

Here the infinum is to be taken over the entire Kähler manifold $\mathcal{M}$ of scalar field values. The key to this section lies in the following simple observation.

## Theorem

If $\exists \phi_0 \in \mathcal{M} : \partial G|_{\phi_0} = 0$ then

(1) $\phi_0$ is a critical point of $V$, $(\partial V|_{\phi_0} = 0)$;

(2) $V|_{\phi_0} \leq 0$.

## Corollary

If $\Lambda = 0$ and supergravity is broken, then $\forall \phi \in \mathcal{M}, \partial G \neq 0$.

## Proof

The proof is trivial. We just observe that

$$\begin{aligned}\delta_i V &= e^G[(\delta_i\delta_j G + \delta_i G\ \delta_j G)\delta^j G - 2\delta_i G] + \tfrac{1}{2}\delta_i(D_\alpha D^\alpha) \qquad (6.3)\\ &= e^G[(\delta_i\delta_j G + \delta_i G\ \delta_j G)\delta^j G - 2\delta_i G] + (\delta_i D_\alpha)D^\alpha - \tfrac{1}{2}(\delta_i f^R{}_{\alpha\beta})D^\alpha D^\beta \qquad (6.4)\end{aligned}$$

and

$$D_\alpha = \delta_i G\ [t_\alpha{}^i{}_j]\ \phi^j. \qquad (6.5)$$

Note that the use of covariant notation has allowed the proof to be derived for noncanonical kinetic energies with essentially no extra work over that which would be required for the canonical case. Observe further that the above theorem shows us that the search for the zeros of $\partial G$ is as important for the breaking of supergravity as is the search for the zeros of $\partial f$ in rigid supersymmetries. In the supergravity case the nonanalyticity of $G$ leads to extra technical difficulties. We shall not pursue this subject, rather we shall prove a sort of converse to the previous theorem.

Let $V_\epsilon$ denote the potential $V$ with all occurrences of $G$ replaced by $\epsilon G$, being careful to remember that the index contraction $\delta_i G \delta^i G$ hides an occurrence of the inverse metric. Thus:

$$V_\epsilon = e^{\epsilon G}\ [\epsilon \delta_i G\ \delta^i G - 3] + \tfrac{1}{2}\ \epsilon^2\ D_\alpha\ D^\alpha. \qquad (6.6)$$

## Theorem

If $\inf(\delta_i G\ \delta^i G) = \eta > 0$, then $\exists \epsilon_0 \in (0, 3/\eta] : V_{\epsilon_0}$ has $\Lambda = 0$.

## Proof

Let $\Lambda_\epsilon = \inf V_\epsilon$. Then

$$\begin{aligned}&1)\quad \lim_{\epsilon\to 0} V_\epsilon = -3, \qquad (6.7)\\ &2)\quad V_{(3/\eta)} \geq \exp\left(\tfrac{3}{\eta}G\right)\ [\tfrac{3}{\eta}\eta - 3] + \tfrac{1}{2}(\tfrac{3}{\eta})^2\ D_\alpha\ D^\alpha \geq 0. \qquad (6.8)\end{aligned}$$

So

$$\begin{aligned}&1)\quad \lim_{\epsilon\to 0} \Lambda_\epsilon = -3, \qquad (6.9)\\ &2)\quad \Lambda_{(3/\eta)} \geq 0. \qquad (6.10)\end{aligned}$$

Since $\Lambda_\epsilon$ is continuous in $\epsilon$ it follows that $\exists \epsilon_0 \in (0, 3/\eta]$ such that $\Lambda_{\epsilon_0} = 0$. Note that $\inf(V_\epsilon)$ is a continuous function of $\epsilon$ even if the location of the absolute minimum is not a continuous function of $\epsilon$. Further observe that in terms of the superpotential $W$ the scaling transformation adopted here is:

$$G \mapsto \epsilon\, G. \tag{6.11}$$

$$K \mapsto \epsilon\, K. \tag{6.12}$$

$$W \mapsto W^\epsilon. \tag{6.13}$$

Some comments on the analytic structure of the superpotential $W$ are in order. It was pointed out by Bagger and Witten [1] that the superpotential $W$ is an analytic section of some holomorphic line bundle constructed over the Kähler manifold $\mathcal{M}$. The word analytic is potentially misleading. What is really required [2] is that $W$ be a function of the $\phi$'s only, not of the $\overline{\phi}$'s [*i.e.* $W = W(\phi)$]. But the superpotential does not have to be everywhere differentiable in order for the Lagrangian to make sense. In particular both poles and branch cuts are permissible, though they may be considered unpleasant. It is useful at this stage to classify superpotentials as follows.

**Class I:** $W$ analytic but not entire on $\mathcal{M}$
—so that poles/branch cuts exist.

**Class II:** $W$ entire but $\ln W$ not entire
—so that $W$ has zeros on $\mathcal{M}$.

**Class III:** $\ln W$ entire on $\mathcal{M}$.

It is common to restrict attention to Class II superpotentials. This would be inappropriate in our discussion since Class II is not closed under the action of the scaling transformation $W \mapsto W^\epsilon$. Indeed, elements of Class II are in general mapped into Class I by this transformation. While commonly occurring superpotentials are of Class II, the other possibilities should not be ignored. In particular, if the Kähler manifold $\mathcal{M}$ is compact [1], then the superpotential is either identically zero or is of Class I.

It is possible to arrange for supergravity breaking with $\Lambda = 0$ by using superpotentials from any of the Classes I, II, or III. This may best be seen by explicit examples.

## Class I

Take

$$\begin{aligned} K &= \overline{\phi}\phi; && (6.14)\\ W &= \phi^{\frac{3}{4}}; && (6.15)\\ G &= \overline{\phi}\phi + \tfrac{3}{4}\ln\overline{\phi}\phi. && (6.16) \end{aligned}$$

A quick computation yields

$$V = \exp(\overline{\phi}\phi)\ \ [\overline{\phi}\phi]^{-\frac{1}{4}}\ (\overline{\phi}\phi - \tfrac{3}{4})^2. \tag{6.17}$$

The absolute minimum occurs at $\phi\overline{\phi} = \frac{3}{4}$, $V = 0$ with $m_{3/2} = e^{G/2} = e^{K/2}|W| = (\frac{3}{4}e)^{\frac{3}{8}}$.

This potential has been discussed by Ferrara *et al.* [10], by Deser and Zumino [11], and by Gaillard *et al.* [12]. We shall later return to this example in a new disguise.

## Class II

Take the Polonyi potential [13]:

$$\begin{aligned} K &= \overline{\phi}\phi; && (6.18)\\ W &= \phi + (2-\sqrt{3}); && (6.19)\\ G &= \overline{\phi}\phi + \ln\{|\phi + (2-\sqrt{3})|^2\}. && (6.20) \end{aligned}$$

## Class III

Take

$$\begin{aligned} K &= \overline{\phi}\phi; && (6.21)\\ \ln W &= \tfrac{1}{2}\phi^2 + i\sqrt{3}\phi; && (6.22)\\ G &= \overline{\phi}\phi + \tfrac{1}{2}\phi^2 + i\sqrt{3}\phi + \tfrac{1}{2}\overline{\phi}^2 - i\sqrt{3}\,\overline{\phi}. && (6.23)\\ \partial G &= \overline{\phi} + \phi + i\sqrt{3}. && (6.24)\\ V &= e^G[|\overline{\phi} + \phi + i\sqrt{3}|^2 - 3] && (6.25)\\ &= e^G(\phi + \overline{\phi})^2. && (6.26) \end{aligned}$$

The absolute minimum occurs at $V = 0$; $\mathrm{Re}\,\phi = 0$; and $\mathrm{Im}\,\phi$ arbitrary, while

$$m_{3/2}{}^2 = e^G = \exp(\tfrac{1}{2}(\mathrm{Re}\,\phi)^2 - 2\sqrt{3}\ \mathrm{Im}\,\phi), \tag{6.27}$$

$$m_{3/2} = \exp(-\sqrt{3}\ \mathrm{Im}\,\phi). \tag{6.28}$$

These examples are sufficient to indicate that all of the Classes I, II, and III are potentially of interest. We shall now leave these toy models and return to a more general analysis. One reason that we have emphasized the different possible analyticity structures of the superpotential $W$ is given by the following theorem.

## Theorem

If $\Lambda = 0$ and supergravity is broken,
then either the superpotential W is not analytic at the origin,
or the model contains at least one gauge singlet superfield.

## Proof

Assume the contrary, that the superpotential is analytic at the origin, and that the model contains no gauge singlet superfields. We shall show that under these conditions either supergravity is unbroken or $\Lambda \neq 0$.

If the superpotential is analytic at zero field, and does not depend on any gauge singlets, then the gauge invariance of the superpotential implies

$$\left.\frac{\partial W}{\partial \phi^i}\right|_{\phi=0} = 0. \tag{6.29}$$

Now noting that $D_\alpha = 0$ at $\phi = 0$ we see that

$$\begin{aligned} V(\phi = 0) &= e^K \left\{(\delta_i W + W\delta_i K)\,(\delta^i \overline{W} + \overline{W}\delta^i K) - 3\,\overline{W}W\right\} \\ &\qquad + \tfrac{1}{2} D_\alpha D^\alpha \qquad (6.30)\\ &= e^K\,\overline{W}\,W\,(\delta_i K\,\delta^i K - 3). \qquad (6.31) \end{aligned}$$

The Kähler potential must itself be differentiable (though certainly not analytic) at the origin. Then gauge invariance of the Kähler potential implies

$$\left.\frac{\partial K}{\partial \phi^i}\right|_{\phi=0} = 0. \tag{6.32}$$

Therefore $V(\phi = 0) = -3\, e^K\, \overline{W}\, W$.
Thus either $\Lambda < 0$ or $\phi = 0$ is a minimum of $V$ that does not break supergravity.

The significance of this theorem is that it informs us that supergravity breaking is always technically ugly. Either the model contains gauge singlet fields, [a fact that is mysterious at best, and at worst neatly foils any attempts at family unification], or, a scarcely more palatable possibility, the superpotential is not analytic at zero field.

It is relatively easy to guarantee a zero Cosmological constant but the price is high. To get $\Lambda = 0$ one need merely construct a nonanalytic real function $G(\phi, \overline{\phi})$ such that $\partial_i G(\phi, \overline{\phi})$ possesses no zeros. [More rigorously: we really want $\inf(\delta_i G\ \delta^i G) \neq 0$]. Having found such a $G$ it can always be tuned to set $\Lambda = 0$. Unfortunately, this is unnatural in two senses:

1. The set of $G$'s leading to $\Lambda = 0$ is of measure zero in the set of all $G$'s.
2. One must live with either gauge singlets or a nonanalytic superpotential.

While some papers have appeared claiming to lift the unnaturalness of the $\Lambda = 0$ condition (*e.g.* [14]), this can only be done by acts of severe violence to the framework we have been discussing.

We shall now exhibit a model that uses the nonanalyticity of its superpotential to simultaneously break supergravity and its gauge symmetry. We shall then turn to a general discussion of models containing gauge singlets.

# Chapter 7

# Nonanalyticity of the Superpotential

In the previous section we argued that supergravity breaking with $\Lambda = 0$ requires either a nonanalytic superpotential, or the existence of gauge singlet fields. However we have not yet exhibited any specific examples of how the nonanalyticity of $W$ allows us to avoid the presence of gauge singlets.

My attention was first drawn to models of this type by the work of S. Rudaz [15]. Rudaz considers a model with

$$W(\phi) = m_{3/2}\, \mu^2\, e^{-a^2/2}\, e^{(\sqrt{3}-1-a)([z/\mu]-a)} \left[\frac{z}{\mu} - a + 1\right], \tag{7.1}$$

$$z = \sqrt{\phi^i\, H_{ij}\, \phi^j}. \tag{7.2}$$

Here $H_{ij}$ is some gauge invariant matrix. Rudaz was able to show that this choice of superpotential simultaneously breaks supergravity and the gauge symmetry and that $\Lambda = 0$ at the minimum. The analysis of the previous section indicates that the branch cut singularity in $z$ is an essential ingredient in this result. We note that

$$\frac{\partial W}{\partial \phi^i} \approx \frac{H_{ij}\phi^j}{\sqrt{\phi^k\, H_{kl}\, \phi^l}} \qquad \text{for } \phi \text{ near zero.} \tag{7.3}$$

The fact that $(\frac{\partial W}{\partial \phi^i})$ is not well behaved as $\phi \to 0$ is what permits us to avoid the use of gauge singlets.

We shall now exhibit a somewhat simpler example that exhibits the same behaviour. The example will be constructed by utilizing a systematic search among all power law superpotentials.

Consider a model of the form

$$K = \overline{\phi}\phi = \phi^{i^*}\phi^i, \tag{7.4}$$
$$W = (\phi\phi)^\gamma = (\phi^i\phi^i)^\gamma \tag{7.5}$$
$$G = \overline{\phi}\phi + \gamma\ln[(\phi\phi)(\overline{\phi\phi})] \tag{7.6}$$

Then

$$\inf(\delta_i G\,\delta^i G) = \inf(|\partial G|^2) \tag{7.7}$$
$$= \inf\left(|\overline{\phi} + \gamma\tfrac{\phi}{(\phi\phi)}|^2\right) \tag{7.8}$$
$$= \inf\left(2\gamma + \overline{\phi}\phi + \gamma^2[\tfrac{(\overline{\phi}\phi)}{(\phi\phi)(\overline{\phi\phi})}]\right) \tag{7.9}$$
$$\geq 2\gamma \tag{7.10}$$

Thus applying the theorems of the previous section we see that:

$\forall\,\gamma > 0$, $\exists\,\epsilon$ such that the model

$$K = \epsilon(\overline{\phi}\phi), \tag{7.11}$$
$$W = (\phi\phi)^{\epsilon\gamma}, \tag{7.12}$$
$$G = \epsilon[(\overline{\phi}\phi) + \gamma\ln[(\phi\phi)(\overline{\phi\phi})]] \tag{7.13}$$

has $\Lambda = \inf V = 0$.

Note that because of the simple form of the Kähler potential we can further simplify this by defining $\phi_{new} = \sqrt{\epsilon}\phi_{old}$. Then defining $\nu = \epsilon\gamma$ ; $\mu = -\nu\ln\epsilon$ we see that the model

$$K = \overline{\phi}\phi, \tag{7.14}$$
$$W = \mu(\phi\phi)^\nu \tag{7.15}$$
$$G = \overline{\phi}\phi + \nu\ln[(\phi\phi)(\overline{\phi\phi})] + 2\mu, \tag{7.16}$$

has $\Lambda = 0$ for at least some choices of $\nu$ and $\mu$.

Let us now evaluate the scalar potential

$$V = e^G\{|\overline{\phi} + 2\nu\tfrac{\phi}{(\phi\phi)}|^2 - 3\} + \tfrac{1}{2}(\overline{\phi}t\phi)^2, \tag{7.17}$$
$$V = e^G\{\overline{\phi}\phi[1 + \tfrac{4\nu^2}{(\phi\phi)(\overline{\phi\phi})}] + 4\nu - 3\} + \tfrac{1}{2}(\overline{\phi}t\phi)^2, \tag{7.18}$$

$$
\begin{aligned}
V &= \{e^G(\phi\phi)^{-1}(\overline{\phi\phi})^{-1}\}\{\overline{\phi}\phi[(\phi\phi)(\overline{\phi\phi}) + 4\nu^2] + [4\nu - 3](\phi\phi)(\overline{\phi\phi})\} \\
&\qquad\qquad\qquad + \tfrac{1}{2}(\overline{\phi}t\phi)^2, \qquad (7.19) \\
V &= \{e^G(\phi\phi)^{-1}(\overline{\phi\phi})^{-1}\} \times \\
&\left\{\overline{\phi}\phi \left| \, |\phi\phi| - 2\nu \right|^2 + 2\nu \left|\phi\sqrt{\overline{\phi\phi}} - \overline{\phi}\sqrt{\phi\phi}\right|^2 + (8\nu - 3)(\phi\phi)(\overline{\phi\phi})\right\} \\
&\qquad\qquad\qquad + \tfrac{1}{2}(\overline{\phi}t\phi)^2. \qquad (7.20)
\end{aligned}
$$

The potential has thus been reduced to a sum of squares. It is now easily seen that $\Lambda = \inf V = 0$ if and only if $\nu = +\frac{3}{8}$.

For $\nu = +\frac{3}{8}$ the model reduces to

$$
\boxed{
\begin{aligned}
&K = \overline{\phi}\phi, \\
&W = (\phi\phi)^{\frac{3}{8}}, \\
&G = \overline{\phi}\phi + \tfrac{3}{8}\ln[(\phi\phi)(\overline{\phi\phi})] \ + 2\mu, \\
&V = e^{2\mu}e^{(\overline{\phi}\phi)}(\phi\phi)^{-\frac{5}{8}}(\overline{\phi\phi})^{-\frac{5}{8}}\left\{ \overline{\phi}\phi \ \left| \, |\phi\phi| - \tfrac{3}{4}\right|^2 + \tfrac{3}{4}\left|\phi\sqrt{\overline{\phi\phi}} - \overline{\phi}\sqrt{\phi\phi}\right|^2 \right\} + \tfrac{1}{2}(\overline{\phi}t\phi)^2.
\end{aligned}
}
\qquad (7.21)
$$

The absolute minimum occurs at

$$
\boxed{
\begin{aligned}
&\text{(a) } |(\phi\phi)| = \tfrac{3}{4} \\
&\text{(b) } \overline{\phi} = \sqrt{\tfrac{(\overline{\phi\phi})}{(\phi\phi)}}\,\phi \\
&\text{(c) } D = 0 \\
&\text{(d) } \Lambda = 0 \\
&\text{(e) } m_{3/2} = e^{G/2} = e^{K/2}|W| = \mu(\tfrac{3}{4}e)^{\frac{3}{8}}
\end{aligned}
}
\qquad (7.22)
$$

Note that the condition $|(\phi\phi)| = \frac{3}{4}$ leaves undecided the direction of the gauge symmetry breaking. Also note that all factors of $M_{Planck}$ have been absorbed into my definition of the field variables. Consequently the physical scale of the gauge symmetry breaking in this model is $M_{Planck}$, while the scale of supergravity breaking is given by the free parameter $m_{3/2}$.

This example shows that it is possible to simultaneously break both supergravity and gauge symmetry at the tree level. This is as far as I wish to pursue this particular avenue and we shall now return to a more general setting to consider the case of models with gauge singlets.

# Chapter 8

# Sector Structure

We shall now turn to the possibility of achieving supergravity breaking by the inclusion of gauge singlets. The analysis so far has avoided making any simplifying assumptions about that structure of the Kähler potential ($K$), the superpotential ($W$), of the gauge metric ($f_{\alpha\beta}$). To leave the model so unconstrained would, at this stage, lead to unmanageable algebraic difficulties. To simplify life we shall assume that the models divide into uncoupled sectors. The different sectors cannot be completely decoupled, since if nothing else, they all couple to gravity. At best we can try to minimize the cross-coupling.

We shall start by assuming that the Kähler manifold describing the scalar fields is a product manifold $\mathcal{M} = \mathcal{M}_1 \otimes \mathcal{M}_2$, and that the metric on $\mathcal{M}$ is the natural one induced from $\mathcal{M}_1 \otimes \mathcal{M}_2$. Splitting the coordinates on $\mathcal{M}$ according to $\phi = (\phi_1, \phi_2)$, this means we may write

$$K(\phi, \overline{\phi}) = K_1(\phi_1, \overline{\phi}_1) + K_2(\phi_2, \overline{\phi}_2), \tag{8.1}$$

$$g_{ij^*} = \begin{bmatrix} g_1 & 0 \\ 0 & g_2 \end{bmatrix}_{ij^*} = g_1 \oplus g_2. \tag{8.2}$$

This further implies

$$D_\alpha = (\partial_i G \, [t_\alpha{}^i{}_j] \, \phi^j) = D_{\alpha,1} + D_{\alpha,2}. \tag{8.3}$$

A subtle point is the choice of condition to be imposed on the superpotential. The best choice appears to be

$$W(\phi) = W_1(\phi_1) \times W_2(\phi_2). \tag{8.4}$$

For this choice

$$G(\phi,\overline{\phi}) = G_1(\phi_1,\overline{\phi}_1) + G_2(\phi_2,\overline{\phi}_2), \tag{8.5}$$

thus leading to a clean separation when $G$ is inserted into the scalar potential. This choice ($W = W_1 W_2$) is the one advocated by Cremmer *et al.* [6]. An alternative choice $W(\phi) = W_1(\phi_1) + W_2(\phi_2)$, championed by Hall *et al.*, fails in its primary objective, that of obtaining a clean separation of the sectors.

There is no particularly appealing choice for the gauge metric ${f^R}_{\alpha\beta}$ and we shall leave it arbitrary. We observe

$$\begin{aligned} V &= e^G\,(\delta_i G\,\delta^i G - 3) + \tfrac{1}{2}\,D_\alpha\,D^\alpha && (8.6)\\ &= e^{G_1+G_2}\,(\delta_i G_1\,\delta^i G_1 + \delta_i G_2\,\delta^i G_2 - 3)\\ &\qquad +\tfrac{1}{2}(D_{\alpha,1} + D_{\alpha,2})\,({D^\alpha}_1 + {D^\alpha}_2) && (8.7)\\ &= e^{G_2}\,[e^{G_1}\,(\delta_i G_1\,\delta^i G_1 - 3) + \tfrac{1}{2} D_{\alpha,1}\,{D^\alpha}_1]\\ &\qquad + e^{G_1}\,[e^{G_2}\,(\delta_i G_2\,\delta^i G_2)] + \tfrac{1}{2}[D_{\alpha,2}\,{D^\alpha}_2 + 2D_{\alpha,1}\,{D^\alpha}_2]. && (8.8)\end{aligned}$$

Our previous arguments have shown that in order to break supergravity and have an analytic superpotential we must have gauge singlets present in the model. Accordingly let us assume that sector 1 consists solely of gauge singlets, while sector 2 may contain both gauge singlets and gauge multiplets. Under this assumption $D_{\alpha,1} = 0$ and we have

$$\begin{aligned} V &= e^{G_2}\,[e^{G_1}(\delta_i G_1\,\delta^i G_1 - 3)]\\ &\qquad +[e^{G_1}\,(e^{G_2}\,\delta_i G_2\,\delta^i G_2) + \tfrac{1}{2}\,D_{\alpha,2}\,{D^\alpha}_2] && (8.9)\\ V &= e^{G_2}\,[V_1] + [V_2]. && (8.10)\end{aligned}$$

Here $V_1$ is just the usual scalar potential for gauge singlets coupled to supergravity while $V_2$ is by construction positive semidefinite. We shall adopt the suggestive nomenclature of calling sector 1 the cosmological sector ($\Lambda$ sector), and calling sector 2 the matter sector.

Indeed let us now write:

$$\begin{aligned} V &= e^{G_m} V_\Lambda + V_m, && (8.11)\\ V_\Lambda &= e^{G_\Lambda}\,(\delta_i G_\Lambda\,\delta^i G_\Lambda - 3), && (8.12)\\ V_m &= e^{G_\Lambda}\,e^{G_m}\,(\delta_i G_m\,\delta^i G_m) + \tfrac{1}{2}\,D_\alpha\,D^\alpha. && (8.13)\end{aligned}$$

Suppose now that the cosmological sector has been chosen so that $V_\Lambda$ has an absolute minimum at ${\phi_\Lambda}^0$ with $V_\Lambda = 0$. Suppose further that the matter sector satisfies $\delta_i G_m = 0$ at ${\phi_m}^0$. Then the point $({\phi_\Lambda}^0, {\phi_m}^0)$ is an absolute minimum of the full scalar potential $V$ with zero cosmological constant. This justifies the terminology "cosmological sector" since it is the cosmological sector that is responsible for setting the cosmological constant equal to zero. It may be tempting to consider renaming the cosmological sector the hidden sector. Resist this temptation. While it may often be the case that the particles in the cosmological sector are very heavy this need not in general be true.

It should be noted that this analysis has provided a very useful constructive technique for building models with zero cosmological constant. One starts with any set of gauge singlets whose mutual interactions satisfy $\Lambda = 0$. (Many such examples are known. For instance, recall the models we exhibited when discussing Class I, II, and III superpotentials.) Now one just pastes on any arbitrary collection of fields such that the equation $\partial_i G_{new fields} = 0$ has one (or more) solutions. Any model constructed in this way will still have $\Lambda = 0$ after inclusion of the new fields.

The most important result of hypothesizing a sector structure as detailed above is that it implies a radical simplification of the mass matrices.

By hypothesis the vacuum in such a sector model satisfies

$$1)\quad V = V_\Lambda = V_m = 0. \tag{8.14}$$

$$2)\quad \delta_i G_m = 0. \tag{8.15}$$

$$3)\quad D_\alpha = 0. \tag{8.16}$$

$$4)\quad \delta_i V = \delta_i V_\Lambda = \delta_i V_m = 0. \tag{8.17}$$

(Naturally these conditions are interrelated.) Consider now the terms contributing to the spin 0 mass matrix

$$\begin{aligned}\delta_i \delta_{j^*} V &= e^{G_m}\,[\delta_i \delta_{j^*} V_\Lambda] + e^G\,\delta_i \delta_{j^*}[\delta_k G_m\,\delta^k G_m] \\ &\qquad + \tfrac{1}{2}\delta_i\delta_{j^*}[D_\alpha\,({f_R}^{-1})^{\alpha\beta}\,D_\beta] \qquad (8.18)\\ &= e^{G_m}\,[\delta_i \delta_{j^*} V_\Lambda] + e^G\,[g_{ij^*} + \delta_i\delta_k G_m\,\delta_{j^*}\delta^k G_m] \\ &\qquad + \delta_i D_\alpha\,({f_R}^{-1})^{\alpha\beta}\,\delta_{j^*} D_\beta. \qquad (8.19)\end{aligned}$$

Now also

$$\begin{aligned}\delta_i \delta_j V &= e^{G_m}\,[\delta_i \delta_j V_\Lambda] + e^G\,\delta_i \delta_j[\delta_k G_m\,\delta^k G_m] \\ &\qquad + \tfrac{1}{2}\delta_i\delta_j[D_\alpha\,({f_R}^{-1})^{\alpha\beta}\,D_\beta] \qquad (8.20)\\ &= e^{G_m}\,[\delta_i \delta_j V_\Lambda] + e^G\,[2\delta_i\delta_j G_m] \\ &\qquad + \delta_i D_\alpha\,({f_R}^{-1})^{\alpha\beta}\,\delta_j D_\beta. \qquad (8.21)\end{aligned}$$

This tells us that the scalar $(mass)^2$ matrix decomposes into a direct sum

$$(m_0)^2 = {(m_0)^2}_\Lambda \oplus {(m_0)^2}_m \tag{8.22}$$

$$\boxed{{(m_0)^2}_\Lambda = e^{G_m} \begin{bmatrix} \delta_{I^*}\delta_J V_\Lambda & \delta_{I^*}\delta_{J^*} V_\Lambda \\ \delta_I \delta_J V_\Lambda & \delta_I \delta_{J^*} V_\Lambda \end{bmatrix}} \tag{8.23}$$

$$\boxed{\begin{aligned} {(m_0)^2}_m = e^G & \begin{bmatrix} \delta_{I^*J} + G_{I^*K^*}G_{KJ} & 2G_{I^*J^*} \\ 2G_{IJ} & \delta_{IJ^*} + G_{IK}G_{K^*J^*} \end{bmatrix} \\ & + \begin{bmatrix} \delta_{I^*}D_A \delta_J D_A & \delta_{I^*}D_A \delta_{J^*} D_A \\ \delta_I D_A \delta_J D_A & \delta_I D_A \delta_{J^*} D_A \end{bmatrix} \end{aligned}} \tag{8.24}$$

where we have used the fact that $D_\alpha = 0$ in the vacuum to write

$$\delta_I D_A = \delta_I ({h_A}^\alpha D_\alpha) = {h_A}^\alpha \, (\delta_I D_\alpha). \tag{8.25}$$

Because $D_\alpha = 0$ at the minimum, the mass matrix for gauge bosons may be written

$$\boxed{{(m_1)^2}_{AB} = 2(\delta_I D_A \, \delta^I D_B).} \tag{8.26}$$

In the fermion sector the "would be Goldstino" now resides solely in the cosmological sector

$$\eta_L = \frac{1}{\sqrt{3}}(\delta_I G_\Lambda \, {\chi_L}^I). \tag{8.27}$$

Thus the fermion mass matrix also decomposes into a direct sum

$$m_{1/2} = m_{1/2_\Lambda} \oplus m_{1/2_m}, \tag{8.28}$$

where

$$\boxed{(m_{1/2_\Lambda})_{IJ} = -e^{G/2}\{{e_I}^i {e_J}^j [\delta_i \delta_j G_\Lambda + \tfrac{1}{3}\delta_i G_\Lambda \delta_j G_\Lambda]\}.} \tag{8.29}$$

The matter spin 1/2 mass matrix mixes gauge non-singlet matter fields with the gauginos

$$\boxed{\begin{aligned} &(m_{1/2_m})_{IJ} = -m_{3/2} \, \delta_I \delta_J G_m \\ &(m_{1/2_m})_{IA} = i\sqrt{2} \, \delta_I D_A \\ &(m_{1/2_m})_{AB} = \tfrac{1}{2} m_{3/2} \, {h_A}^\alpha \, {h_B}^\beta \, df_{\alpha\beta}. \end{aligned}} \tag{8.30}$$

Where $df_{\alpha\beta}$ now only picks up a contribution from the cosmological sector

$$df_{\alpha\beta} = (\delta^i G_\Lambda)(\delta_i f_{\alpha\beta}). \tag{8.31}$$

This direct sum structure for the $(mass)^2$ matrices means that we can construct separate $(mass)^2$ sum rules for the cosmological sector and the matter sector.

$$\boxed{\begin{aligned} \mathrm{Str}\,(P_\Lambda m^2) &= \mathrm{tr}\,(m_0)^2{}_\Lambda - 2\,\mathrm{tr}\,(m_{1/2})^2{}_\Lambda - 4{m_{3/2}}^2 \\ &= 2{m_{3/2}}^2[(n_\Lambda - 1) + \delta^i G_\Lambda\; R_i{}^j\; \delta_j G_\Lambda]. \end{aligned}} \tag{8.32}$$

In the matter sector we see

$$\mathrm{tr}\,(m_0)^2{}_m = 2{m_{3/2}}^2[n_m + \delta_i\delta_j G\;\delta^i\delta^j G] + \mathrm{tr}\,(m_1)^2. \tag{8.33}$$

$$\mathrm{tr}\,(m_{1/2})^2{}_m = {m_{3/2}}^2[\delta_i\delta_j G\;\delta^i\delta^j G + \mathrm{tr}\,(df_R\; {f_R}^{-1}\; \overline{d}f_R\; {f_R}^{-1})] + 2\,\mathrm{tr}\,(m_1)^2. \tag{8.34}$$

$$\boxed{\begin{aligned} \mathrm{Str}\,(P_m\, m^2) &= \mathrm{tr}\,(m_0)^2{}_m - 2\,\mathrm{tr}\,(m_{1/2})^2{}_m + 3\,\mathrm{tr}\,(m_1)^2, \\ &= 2{m_{3/2}}^2[n_m - \mathrm{tr}\,(df_R\; {f_R}^{-1}\; \overline{d}f_R\; {f_R}^{-1})]. \end{aligned}} \tag{8.35}$$

Indeed the restrictions implied by the assumed sector structure allow us to go even further in the reduction of the mass matrices.

# Chapter 9

# The Matter Sector: Leptoquarks and Higgses

The matter sector by construction contains both the leptoquark and Higgs fields of the model. Let us define the leptoquarks as those superfields whose scalar components do not acquire a gauge symmetry breaking vacuum expectation value. All other superfields (including gauge singlets) will be called Higgs fields. Consider the object $\delta_i D_\alpha = g_{ij^*}[t_\alpha{}^{j^*}{}_{k^*}]\phi^{k^*}$. Then by definition, $\delta_i D_\alpha = 0$ for the leptoquarks. Thus we see that among the spin 1/2, fields leptoquarks do not mix with gauginos, though in general Higgsinos do mix with gauginos. For the scalar $(mass)^2$ matrix we see that at least the gauge contribution does not mix sleptosquarks with Higgses. Indeed, we see that if any sleptosquark Higgs mixing, or leptoquark Higgsino mixing does occur, this mixing can only arise from the terms involving $\delta_i\delta_j G_m$.

We shall now assume that $\delta_i\delta_j G_m$ is a direct sum

$$\delta_i\delta_j G_m = (\delta_i\delta_j G)_{LQ} \oplus \delta_i\delta_j G_H. \tag{9.1}$$

A decomposition of this type could certainly be achieved if the matter sector itself had sector structure $(K_m = K_{LQ} + K_H; W_m = W_{LQ}W_H)$. The sector structure hypothesis is unfortunately too strong since in such a case leptoquark masses are independent of the Higgs vacuum expectation values. Thus leptoquark multiplets get gauge invariant masses. To avoid this problem, a sufficiently general ansätz is

$$G_m = G_{LQ} + G_H + G_{mix} \tag{9.2}$$

where $G_{mix}$ is at least quadratic in leptoquark fields. An even more restrictive ansätz is

$$K_m = K_{LQ} + K_H \tag{9.3}$$

$$W_m = W_{LQ} W_H e^{\Omega} \tag{9.4}$$

where $\Omega$, taken to be at least quadratic in leptoquark fields, is responsible for the mass splittings within leptoquark multiplets. However, it should be noted that the only assumption that is really necessary is that $\delta\delta G_m$ decomposes into a direct sum when evaluated at the vacuum. Under this condition, the matter sector mass matrices themselves decompose into direct sums:

$$\begin{array}{l} {(m_0)^2}_m = {(m_0)^2}_{LQ} \oplus {(m_0)^2}_H, \\ (m_{1/2})_m = (m_{1/2})_{LQ} \oplus (m_{1/2})_H. \end{array} \tag{9.5}$$

It is now easy to see that

$$\begin{aligned} {(m_0)^2}_{LQ} &= {m_{3/2}}^2 \begin{bmatrix} \delta_{I^*J} + (\delta_J\delta_K G)_{LQ}(\delta^K\delta_{I^*}G)_{LQ} & 2(\delta_{I^*}\delta_{J^*}G)_{LQ} \\ 2(\delta_I\delta_J G)_{LQ} & \delta_{IJ^*} + (\delta_I\delta_K G)_{LQ}(\delta^K\delta_{J^*}G)_{LQ} \end{bmatrix} && (9.6) \\ &= {m_{3/2}}^2 \begin{bmatrix} \delta_{I^*K} & (\delta_{I^*}\delta_{K^*}G)_{LQ} \\ (\delta_I\delta_K G)_{LQ} & \delta_{IK^*} \end{bmatrix} \begin{bmatrix} \delta_{K^*J} & (\delta_{K^*}\delta_{J^*}G)_{LQ} \\ (\delta_K\delta_J G)_{LQ} & \delta_{KJ^*} \end{bmatrix} && (9.7) \end{aligned}$$

The scalar mass matrix is

$$\boxed{(m_0)_{LQ} = m_{3/2} \begin{bmatrix} \delta_{I^*J} & (\delta_{I^*}\delta_{J^*}G)_{LQ} \\ (\delta_I\delta_J G)_{LQ} & \delta_{IJ^*} \end{bmatrix}} \tag{9.8}$$

For the spin 1/2 particles

$$\boxed{(m_{1/2})_{LQ} = -m_{3/2}[(\delta_I\delta_J G)_{LQ}]} \tag{9.9}$$

Warning: $(\delta_I\delta_J G)_{LQ} = \delta_I\delta_J(G_{LQ} + G_{mix})$. The Higgs sector has not been improved by our ansatz. Indeed,

$$\boxed{{(m_0)^2}_H = {m_{3/2}}^2 \left\{ \begin{bmatrix} \delta_{I^*J} & \delta_{I^*}\delta_{J^*}G_H \\ \delta_I\delta_J G_H & \delta_{IJ^*} \end{bmatrix}^2 \right\} + \begin{bmatrix} (\delta_{I^*}D_A\delta_J D_A) & (\delta_{I^*}D_A\delta_{J^*}D_A) \\ (\delta_I D_A\delta_J D_A) & (\delta_I D_A\delta_{J^*}D_A) \end{bmatrix}} \tag{9.10}$$

$$\boxed{(m_{1/2})_H = \left[ \begin{array}{cc} -m_{3/2}\delta_I\delta_J G_H & i\sqrt{2}\delta_I D_A \\ i\sqrt{2}\delta_J D_B & m_{3/2} h_A{}^\alpha h_B{}^\beta df^R{}_{\alpha\beta} \end{array} \right]} \tag{9.11}$$

While the masses in the Higgs sector are in general quite complicated the masses in the leptoquark sector are now easily diagonalized.

Since $(\delta_I\delta_J G)$ is a complex symmetric matrix it may be decomposed as follows

$$(\delta_I\delta_J G) = (U\mu U^T)_{IJ}. \tag{9.12}$$

Here $U$ is a unitary matrix, $U^T$ is its transpose , and $\mu$ may be chosen to be a real, positive semidefinite and diagonal matrix.

Now observe that

$$\left[ \begin{array}{cc} I & \overline{\delta\delta G} \\ \delta\delta G & I \end{array} \right] = \left[ \begin{array}{cc} I & \overline{U\mu U^T} \\ U\mu U^T & I \end{array} \right] = \left[ \begin{array}{cc} I & \overline{U}\mu U^{-1} \\ U\mu U^T & I \end{array} \right]$$
$$= \left[ \begin{array}{cc} \overline{U} & 0 \\ 0 & U \end{array} \right] \left[ \begin{array}{cc} I & \mu \\ \mu & I \end{array} \right] \left[ \begin{array}{cc} U^T & 0 \\ 0 & U^{-1} \end{array} \right]. \tag{9.13}$$

So diagonalizing $(m_{1/2})_{LQ}$ leads to

$$(m_{1/2})_{LQ} = -m_{3/2}\mu,$$
$$(m_0)_{LQ} = m_{3/2} \left[ \begin{array}{cc} I & \mu \\ \mu & I \end{array} \right]. \tag{9.14}$$

The eigenvalues of the scalar mass matrix are seen to be

$$\lambda^{\pm i} = m_{3/2}(1 \pm \mu^i). \tag{9.15}$$

In terms of tree level masses we now see

$$\boxed{(m_0{}^{\pm i})_{LQ} = |m_{3/2} \pm (m_{1/2}{}^i)_{LQ}|} \tag{9.16}$$

This sum rule now connects the masses of the leptons and quarks with those of the sleptons and squarks. This sum rule has previously been discussed by Cremmer *et al.* [6], but emerges in this context in a more general framework. In particular the analysis presented here does not require canonical kinetic energy terms for leptons and quarks. This sum rule is rather robust, the technical assumptions we have made to enable derivation of this sum rule may be summarized as:

- 1) $G_{Total} = G_\Lambda + G_{Higgs} + G_{leptoquark} + G_{mix}$.
- 2)
  - (a) $G_\Lambda$ is chosen so that $\Lambda = 0$.
  - (b) $\delta_i G_{Higgs} = 0$ has a solution at ${\phi^i}_{Higgs} \neq 0$.
  - (c) $\delta_i G_{leptoquark} = 0$ at ${\phi^i}_{leptoquark} = 0$.

An approximate sum rule may be obtained for the Higgs sector. The derivation we used in the leptoquark sector is spoiled in the Higgs sector by terms proportional to $\delta_I D_A$. Thus we may write

$$\boxed{({m_0}^{\pm i})_H = |m_{3/2} \pm ({m_{1/2}}^i)_H| + o(m_{gauge}).} \tag{9.17}$$

This approximate sum rule relates the masses of the scalar Higgs particles to those of their associated Higgsinos. Unfortunately, in realistic models $m_{gauge}$ is likely to be of order $m_{3/2}$ or of order $m_{GUT}$, so that this sum rule is likely to be very badly broken.

This completes our analysis of the matter sector.

# Chapter 10

# Masses in the Cosmological Sector

In this section we shall only be discussing the cosmological sector, so we may without ambiguity drop the subscript $\Lambda$. The cosmological sector by hypothesis has no gauge interactions, while its vacuum occurs at $V = 0$, $\delta_i V = 0$. These constraints imply that

$$\text{(a)} \qquad G_I\, G^I = 3, \tag{10.1}$$

$$\text{(b)} \qquad G_{IJ}\, G^J = -G_I. \tag{10.2}$$

These constraints have not yet been utilized to their fullest extent. Explicitly evaluating our general formulae for the scalar $(mass)^2$ matrix leads to

$$(m_0)^2 = \left[ \begin{array}{cc} (m_0)^2{}_{I^*J} & (m_0)^2{}_{I^*J^*} \\ (m_0)^2{}_{IJ} & (m_0)^2{}_{IJ^*} \end{array} \right] \tag{10.3}$$

$$(m_0)^2{}_{I^*J} = m_{3/2}{}^2[\delta_{I^*J} + G_{I^*K^*}G_{KJ} + R_{I^*JK}{}^L\, G^K\, G_L] \tag{10.4}$$

$$(m_0)^2{}_{IJ} = m_{3/2}{}^2[2G_{IJ} + G_{IJK}G^K] \tag{10.5}$$

$$\boxed{\begin{aligned} (m_0)^2 = m_{3/2}{}^2 & \left[ \begin{array}{cc} \delta_{I^*K} & G_{I^*K^*} \\ G_{IK} & \delta_{IK^*} \end{array} \right] \left[ \begin{array}{cc} \delta_{K^*J} & G_{K^*J^*} \\ G_{KJ} & \delta_{KJ^*} \end{array} \right] \\ + m_{3/2}{}^2 & \left[ \begin{array}{cc} R_{I^*JK}{}^L G^K G_L & G_{I^*J^*K^*}G^{K^*} \\ G_{IJK}G^K & R_{J^*IK}{}^L G^K G_L \end{array} \right] \end{aligned}} \tag{10.6}$$

The fermion masses are

$$\boxed{m_{1/2\,IJ} = m_{3/2}[\delta_I \delta_J G + \tfrac{1}{3} G_I\, G_J]} \tag{10.7}$$

These matrices may be partially diagonalized. Let the index $I$ run from 0 to $n-1$. Then the vielbein $e_I{}^i$ may be chosen in such a manner that $G_I$ lies along the 0 direction and is real. That is:

$$G_I = \sqrt{3}\,\delta_{0I} = G^I = G_{I^*}. \tag{10.8}$$

This implies

$$\begin{aligned} G_{00} &= -1 && (10.9)\\ G_{0I} &= 0, \qquad I \in [1, 2, \ldots, (n-1)] && (10.10)\\ G_{IJ} & \qquad \text{arbitrary}, \quad I, J \in [1, 2, \ldots, (n-1)] && (10.11) \end{aligned}$$

Now diagonalize $G_{IJ}$, using $G_{IJ} = (U\mu U^T)_{IJ}$, where $\mu$ is real and diagonal and we may choose

$$\begin{aligned} \mu_{IJ} &= \mu_I\,\delta_{IJ} \quad \text{(no summation)}, && (10.12)\\ \mu_{00} &= \mu_0 = -1; \quad \text{all other } \mu_I \text{ positive semidefinite.} && (10.13) \end{aligned}$$

There are $n-1$ physical fermions after elimination of the "would be Goldstino", and their masses are:

$$(m_{1/2})_I = m_{3/2}|\mu_I|. \qquad I \in [1, 2, \ldots, (n-1)]. \tag{10.14}$$

There are $2n$ scalars whose mass matrix reads

$$(m_0)^2 = m_{3/2}{}^2 \left\{ \begin{bmatrix} I & \mu \\ \mu & I \end{bmatrix}^2 + \begin{bmatrix} 3R_{IJ^*00^*} & \sqrt{3}G_{I^*J^*0^*} \\ \sqrt{3}G_{IJ0} & 3R_{JI^*00^*} \end{bmatrix} \right\}. \tag{10.15}$$

If we assume that the contributions from the Riemann tensor and from $G_{IJK}$ are small, then the scalar masses are

$$(m_0)^{\pm I} = m_{3/2} \left| 1 \pm |\mu^I| \right| + o(R, G_{IJK}). \tag{10.16}$$

In particular

$$\boxed{\begin{aligned} (m_0)^{-0} &= o(R, G_{IJK}),\\ (m_0)^{+0} &= 2m_{3/2} + o(R, G_{IJK}). \end{aligned}} \tag{10.17}$$

While for $I$ running from 1 to $n-1$:

$$\boxed{(m_0)^{\pm I} = |m_{3/2} \pm m_{1/2}{}^I| + o(R, G_{IJK}).} \tag{10.18}$$

Note that if both $R_{IJ^*00^*}$ and $G_{IJ0}$ are zero at the minimum, then this predicts the existence of an exactly massless scalar in the cosmological sector. Indeed, particles in the cosmological sector are not necessarily heavy; it is in general misleading to refer to the cosmological sector as a hidden sector.

The Class III example previously considered may be used to illustrate this phenomena. Consider the 1-dimensional model defined by

$$
\begin{aligned}
K &= \overline{\phi}\phi; \qquad \ln W = \tfrac{1}{2}\phi^2 + i\sqrt{3}\phi; && (10.19)\\
G &= \overline{\phi}\phi + \tfrac{1}{2}\phi^2 + i\sqrt{3}\phi + \tfrac{1}{2}\overline{\phi}^2 - i\sqrt{3}\,\overline{\phi}. && (10.20)\\
V &= e^G(\phi + \overline{\phi})^2. && (10.21)
\end{aligned}
$$

The scalar potential is

$$V = \{\exp(2[\,\mathrm{Re}\,\phi]^2 - \sqrt{3}\ \mathrm{Im}\,\phi)\}[2\,\mathrm{Re}\,\phi]^2. \qquad (10.22)$$

Then

$$
\begin{aligned}
&m(\,\mathrm{Re}\,\phi) = 2m_{3/2},\\
&m(\,\mathrm{Im}\,\phi) = 0. && (10.23)
\end{aligned}
$$

Returning to our general analysis, it must be emphasized that the corrections to our approximate mass spectrum [ $\mathrm{o}(R, G_{IJK})$ ] are typically large, often being so large that the approximate spectrum is not useful.

To complete the analysis there is one special case that is amenable to further processing. Let us assume that the cosmological sector is minimal in the sense that it is of complex dimension 1. In this case no spin 1/2 particles remain after elimination of the "would be Goldstino". The Riemann tensor has only one nonzero component.

$$R_{00^*00^*} = R_{00^*} = R. \qquad (10.24)$$

The scalar $(mass)^2$ matrix is now

$$(m_0)^2 = {m_{3/2}}^2 \begin{bmatrix} 2 + 3R & -2 + \sqrt{3}(G_{000})^* \\ -2 + \sqrt{3}G_{000} & 2 + 3R \end{bmatrix}. \qquad (10.25)$$

So

$$\boxed{\mathrm{tr}\,(m_0)^2 = (4 + 6R){m_{3/2}}^2,} \qquad (10.26)$$

while the masses themselves are

$$\boxed{{m_0}^{\pm} = m_{3/2}\sqrt{(2 + 3R) \pm |2 - \sqrt{3}G_{000}|}.} \qquad (10.27)$$

This now completes our analysis of masses in the cosmological sector.

# Chapter 11

# Conclusion

We have investigated the vacuum structure of N=1 supergravity coupled to gauged chiral matter with general noncanonical kinetic energies for both matter fields and gauge fields. We have explicitly calculated the tree level $(mass)^2$ matrices , and have seen how the supertrace of the $(mass)^2$ matrix is affected by noncanonical kinetic energies. The sum rule relating Lepton and Quark masses to those of their scalar partners ($m_0^{\pm} = \left| m_{3/2} \pm m_{1/2} \right|$) was derived in this more general context and so holds even for noncanonical kinetic energies. Some general theorems on the occurrence of supergravity breaking were established. In particular, attention was drawn to the crucial role played by the analyticity of the superpotential at zero field.

# References


[ 1 ] J. Bagger, Nucl. Phys. B211 (1983) 302.
a shorter discussion without gauge interactions may be found in:
E. Witten and J. Bagger, Phys. Lett. 115B (1982) 202.

[ 2 ] E. Cremmer, S. Ferrara, L. Girardello, and A. van Proeyen, Nucl. Phys. B212 (1983) 413.

[ 3 ] L. Hall, J. Lykken, and S. Weinberg, Phys. Rev. D27 (1983) 2359.

[ 4 ] J. Ellis, J. Hagelin, D. Nanopoulos, and K. Tamvakis, Phys. Lett. 125B (1983) 275.

[ 5 ] P. Nath, R. Arnowitt, and A. Chamseddine, Phys. Lett. 121B (1983) 33.

[ 6 ] E. Cremmer, P. Fayet, and L. Girardello, Phys. Lett. 122B (1983) 41.

[ 7 ] M. Claudson, L. Hall, and I. Hinchliffe, Nucl. Phys. B228 (1983) 501.

[ 8 ] S. Goldberg, Curvature and Homology, (Dover,New York,1982).

[ 9 ] E. Flaherty, Hermitian and Kählerian Geometry in Relativity, Lecture Notes in Physics, Vol 46, (Springer–Verlag,Berlin,1976).

[ 10 ] S. Ferrara, D. Freedman, and P. van Nieuwenhuizen, Phys. Rev. D13 (1976) 3214.

[ 11 ] S. Deser and B. Zumino, Phys. Lett. 62B (1976) 335.

[ 12 ] M. Gaillard, L. Hall, B. Zumino, F. del Aguila, J. Polchinski, and G.Ross, Phys. Lett. 122B (1983) 335.

[ 13 ] J. Polonyi, Budapest preprint KFKI–1977–93 (1977).

[ 14 ] E. Cremmer, S. Ferrara, C. Kounnas, and D. Nanopoulos, Phys. Lett. 133B (1983) 61.

[ 15 ] S. Rudaz, unpublished.