Optimal balance of noise sharing and Mexican Hat coupling in a stochastic neural field

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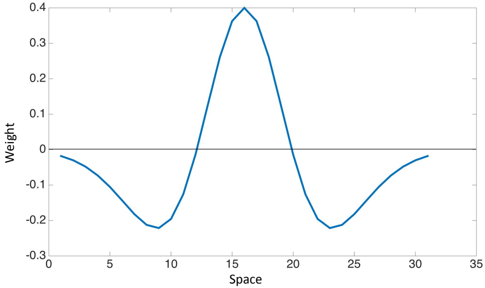
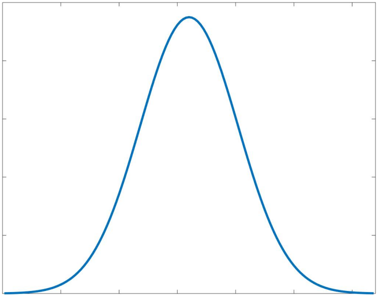
Mexican Hat coupling and noise sharing



Mexican Hat w(x)



Noise sharing h(x)

Stochastic neural field equation

$$dY(t,x) = \left[-Y(t,x) + \int_0^L cw(x-y)Y(t,y)dy \right] dt + \sigma dG(t,x), \quad (1)$$

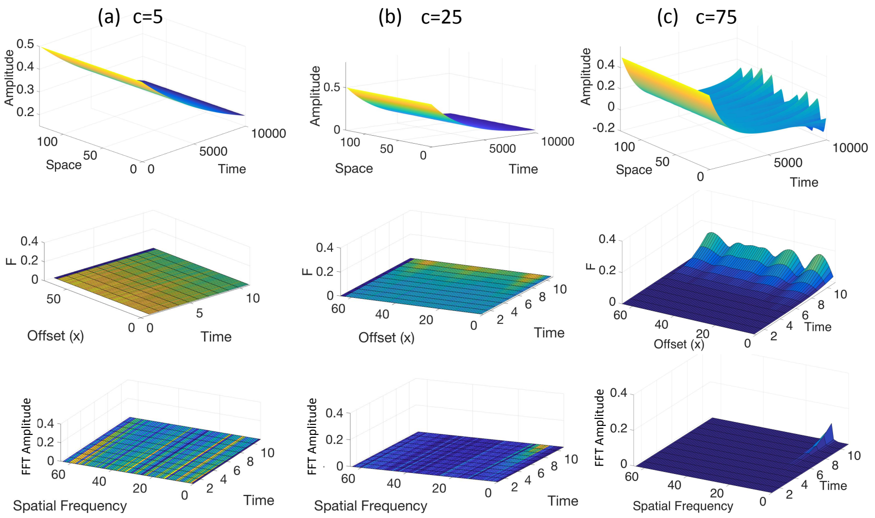
$$w(x) = b_1 \exp(-(x/d_1)^2) - b_2 \exp(-(x/d_2)^2) \quad (2)$$

$$r(x) = \frac{1}{2\eta\sqrt{\pi}} \exp(-x^2/4\eta^2). \quad (3)$$

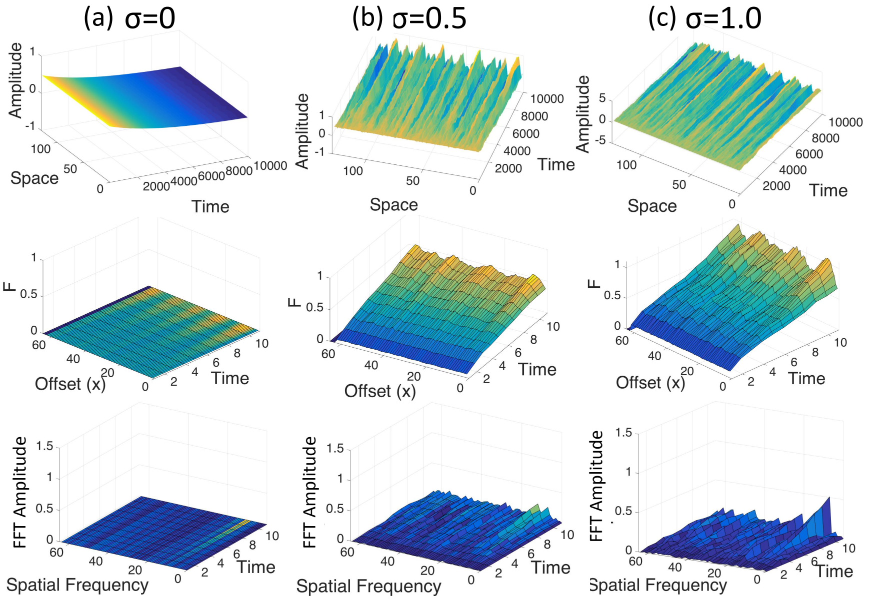
$$G(t,x) = \int_{\mathbb{R}} Z(t,y)h(x-y)dy \quad (4)$$

where

$$r = h * h$$



(a) c=5
(b) c=25
(c) c=75
Amplitude
Space
Time
F
Offset (x)
FFT Amplitude
Spatial Frequency



(a) σ=0
(b) σ=0.5
(c) σ=1.0
Amplitude
Space
Time
F
Offset (x)
Time
FFT Amplitude
Spatial Frequency
Time

Y(t,x), the solution of (1), can be written as

$$Y(t,x) = a_0(t) + \sum_{k\geq 1} 2\mathrm{Re}\left(a_k(t)e^{2\pi ikx/L}\right) \tag{5}$$

where the scalar process $a_0(t)$ satisfies

$$da_0(t) = [-1 + cW(0)]\, a_0(t)\, dt + \frac{\sigma}{\sqrt{L}}\sqrt{R(0)}\, dC_0(t) \tag{6}$$

and for $k \geq 1$ the complex processes $a_k(t)$ satisfy

$$da_k(t) = [-1 + cW(2\pi k/L)]\, a_k(t)\, dt + \frac{\sigma}{\sqrt{2L}}\sqrt{R(2\pi k/L)}\, dC_k(t), \tag{7}$$

where

$$W(k) = \int_0^L e^{-ikx} w(x)\, dx \tag{8}$$

$$R(k) = \int_0^L e^{-ikx} r(x)\, dx \tag{9}$$

with k replaced by $2\pi k/L$.

Write $a_k(t) = A_k(t)e^{i\phi_k(t)}$. Then

$$Y(t,x) = a_0(t) + 2\sum_{k\geq 1} A_k(t)\cos\big(2\pi kx/L + \phi_k(t)\big). \tag{10}$$

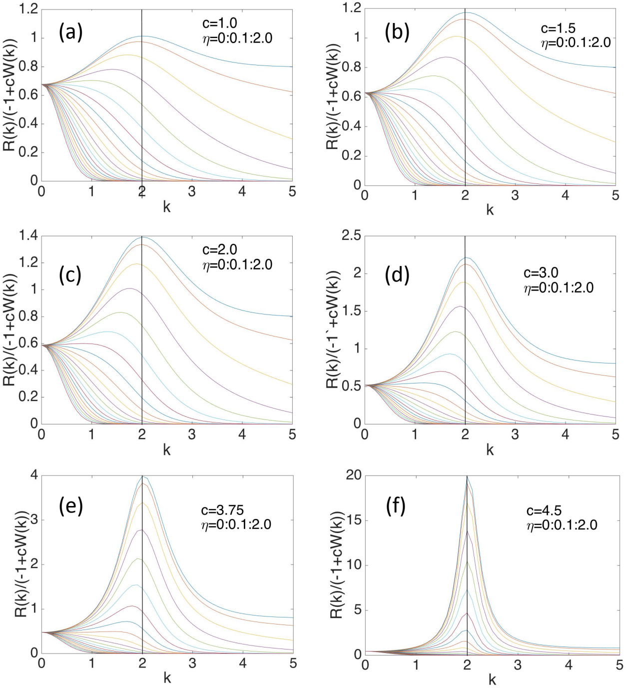
$A_k(t)$ is the amplitude of the O.U. a_k:

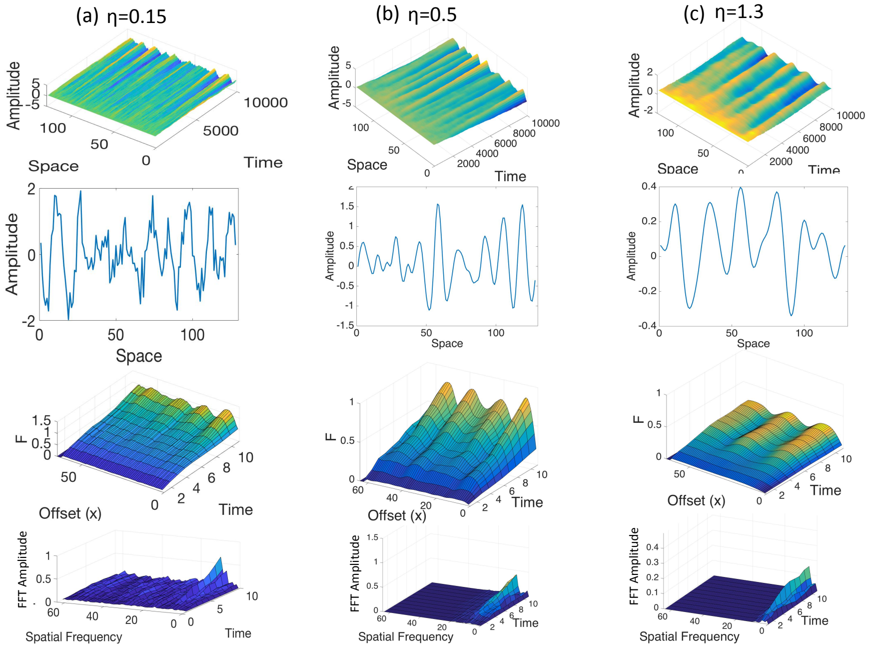
$$da_k(t) = \lambda_k a_k(t)\,dt + \sigma_k\,dC_k(t), \tag{11}$$

where $\lambda_k = -1 + cW(2\pi k/L)$ and $\sigma_k = \frac{\sigma}{\sqrt{2L}}\sqrt{R(2\pi k/L)}$.

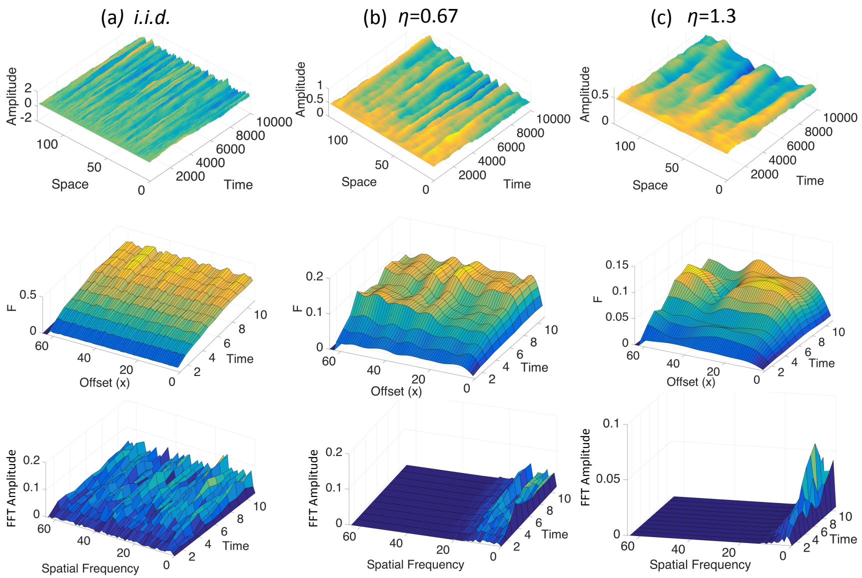
$$E[A_k(t)^2] = e^{2\lambda_k t}A_k(0)^2 + \frac{\sigma_k^2(e^{2\lambda_k t} - 1)}{\lambda_k}. \tag{12}$$

$$argmax(E[A_k(t)^2]) = argmax\left[\frac{R(2\pi k/L)}{(1 - cW(2\pi k/L))}\right] \tag{13}$$


(a)
c=1.0
η=0:0.1:2.0
R(k)/(-1+cW(k))
k
(b)
c=1.5
η=0:0.1:2.0
R(k)/(-1+cW(k))
k
(c)
c=2.0
η=0:0.1:2.0
R(k)/(-1+cW(k))
k
(d)
c=3.0
η=0:0.1:2.0
R(k)/(-1`+cW(k))
k
(e)
c=3.75
η=0:0.1:2.0
R(k)/(-1+cW(k))
k
(f)
c=4.5
η=0:0.1:2.0
R(k)/(-1+cW(k))
k



(a) η=0.15
(b) η=0.5
(c) η=1.3
Amplitude
Space
Time
Offset (x)
F
FFT Amplitude
Spatial Frequency



(a) i.i.d.
(b) η=0.67
(c) η=1.3
Amplitude
Space
Time
F
Offset (x)
FFT Amplitude
Spatial Frequency

Thank you!