



Workshop  
in  
Probability Theory and Mathematical Statistics

30 November–1 December, 2010

Victoria University of Wellington  
Kelburn Campus, Alan MacDiarmid Building, AM 101

**Presenters, titles and abstracts**

(Ordered alphabetically, by presenters' last names)

*A network with two parallel batch service queues*

Lisa Chen, PhD student  
The University of Auckland

A network with two parallel batch service queues, each served by infinitely many servers is considered here. Each queue commences service when a batch is full. We use stochastic comparison and coupling arguments to show that a monotone user equilibrium policy exists and is unique. Moreover, we propose an iterative algorithm for finding the user equilibrium policy.

*Existence and Percolation for Germ-Grain Models from lilypond growth protocols*

Daryl Daley  
Melbourne University

Each grain in a germ–grain model produced via the lilypond protocol grows from its germ until it first makes contact with another grain, when its growth ceases. By studying the possible growth  $d_{jk}$  from grain  $j$  to another grain  $k$ , the ultimate size  $y_j$  of grain  $j$  is expressed as  $\inf_k \{d_{jk} : \text{conditions on } k\}$ , for a variety of simpler models the existence of a unique solution  $y_j$  has been shown. Percolation questions are harder. Any given grain belongs to a cluster via contact with some other grains, whether the contact is direct or via other grains; if there is any cluster of infinite size then the model percolates. Under “ordinary” conditions one expects percolation to be absent, but less has been proven.

*Stochastic representation method in multivariate statistical analysis*

Kai-Tai Fang

BNU-HKBU United International College

Methodology is always important issue in theory development. Prof. P.L. Hsu, who was Professor of Peking University and gave a significant contribution to probability and statistics, told us that there are three main methods in statistics study:

- (a) analytic method including using mathematical analysis, characteristic function, moment generating function, and so on;
- (b) algebraic method including using matrix analysis, transformation Jacobian, group theory, and so on; and
- (c) probability method or stochastic representation method that directly deals with the random variables instead of treating their distribution functions, density functions, or characteristic functions.

Prof. P.L. Hsu said : “The probability method in general is the best among the three methods if it can be applied.” There are many difficult problems in multivariate analysis, especially, in multivariate analysis under the elliptical population (also called generalized multivariate analysis). “Stochastic representation method” (SRM) has played an important role in developing theory of generalized multivariate analysis. In this talk I firstly introduce the SRM in general. Secondly, some basic properties of SRM will be given. Finally for demonstrating powerfulness I show how to use SRM to solve several difficult problems in multivariate statistical analysis.

*On some geometric stochastic differential equations and applications in Micromagnetism*

Benjamin Goldys

The University of New South Wales

Recently, there is a growing interest in stochastic perturbations of deterministic PDEs that describe properties of Riemannian manifolds. Motivations come from certain geometric problems arising in the theory of manifolds and from applications in Physics. In this talk we will present an example of such an equation arising in the theory of harmonic maps. Applications to the theory of magnetic materials will be also discussed.

*Sampling from a very large set defined by a complex system of linear constraints*

Martin Hazelton

Massey University

Motivated by a statistical linear inverse problem from the field of transportation science, we consider sampling from a very large set defined by a complex system of linear constraints. We discuss some componentwise samplers, and show why they can fail completely. We then consider a particular version of the problem (based on estimation of travel rates for linear networks) where a Markov process can generate feasible candidate samples.

*On differentiability of implicitly defined function  
in semi-parametric profile likelihood estimation*

Yuichi Hirose

Victoria University of Wellington

Abstracts: The object of talk is the differentiability of implicitly defined functions which we encounter in the profile likelihood estimation of parameters in semi-parametric models. Scott and Wild (1997, 2001) and Murphy and Vaart (2000) developed methodologies that can avoid dealing with such implicitly defined functions by reparametrizing parameters in the profile likelihood and using an approximate least favorable submodel in semi-parametric models. Our result shows applicability of an alternative approach developed in Hirose (2010) which uses the differentiability of implicitly defined functions.

*Combinatorics and non-monotone random walk couplings*

Mark Holmes

The University of Auckland

Given a stack of arrows (pointing left or right) above each site in  $\mathbb{Z}$ , we can define a walk  $L$  that always steps according to the first unused arrow at its current location. Switching some left arrows to right arrows we get a new walk  $R$ . What is the relationship between the walks  $L$  and  $R$ ?

When the arrows are generated randomly, the answer(s) to the above question enable us to obtain new results for some (non-Markovian) random walk models.

*On occupancy problem in high-dimensional space*

Estate V. Khmaladze

Victoria University of Wellington

We consider occupancy problem for triangular array of multinomial distributions in  $d$ -dimensional space, when the dimension increases and probabilities of individual events change along with the same size  $n$ .

We will describe the asymptotic behavior of the number of events  $\mu_n(k)$  observed exactly  $k$  times,  $k = 1, 2, \dots$ , and the number of different observed events  $\mu_n$ .

The technique we will use to tackle the problem is connected with asymptotic theory for likelihood ratio, contiguity theory and large deviations.

*Multivariate version of the Cramér-von Mises statistic*

Gennady Martynov

Institute for Information Transmission Problems of the Russian Academy of Sciences

We present multivariate version of the Cramér-von Mises statistic. It is the generalization of the classical univariate Cramér-von Mises statistic. The methods are analogous to the methods, presented in Krivyakov, Martynov, Tyurin (1977). We will consider weighted statistics with the power weight functions. The calculation methods for searching eigen-

values and eigenfunctions of the corresponding covariance operators are considered.

*Applications of Kusuoka Representation of Risk Measures*

Spiridon Penev

The University of New South Wales

We investigate a new family of coherent risk measures, indexed by a parameter  $p > 1$ , which are tailored towards applications in Finance. The measures are represented as suprema of integrals of the well-known Average Value of Risk, with respect to a suitable convex set of probability measures on the interval  $(0, 1]$ . Such representations are called Kusuoka representations. Interestingly, when  $p = 2$  these measures relate to the Fano factor in statistics, which is a specific expression for a noise-to-signal ratio.

We also discuss inference about these risk measures. The inference problem turns out to be equivalent to minimizing a convex function subject to convex constraints and can be dealt with.

This is a joint work with D. Dentcheva and A. Ruszczyński.

*Birth and death processes in molecular evolution and biodiversity conservation*

Mike Steel

University of Canterbury

The Yule model (linear birth process) is a popular model for describing the evolution of species from a common ancestor, in which the present-day species appear as the leaves (tips) of this tree. This leads to two interesting questions in mathematical probability:

1. If we observe a binary state at each leaf of the tree, and these states have evolved on the tree from an unknown ancestral state at the root node, under what conditions can we accurately infer this original state?

2. If we delete the tips randomly from the tree under various 'field of bullets' extinction models, how much of the 'evolutionary heritage' described by the tree is likely to be lost?

I describe some of recent our results on these two problems, which are motivated by basic questions in molecular evolution, and biodiversity conservation, respectively.

*Cornish-Fisher expansions for sample autocovariances  
and other functions of sample moments of linear processes*

Kit Withers

Industrial Research Ltd

We give Cornish-Fisher expansions for smooth functions of the sample cross-moments of a stationary linear process. Examples include the distributions of the sample mean, the sample autocovariance and the sample autocorrelation.

*Asymptotic Properties of Partial Sum Processes of Divisible Statistics  
in Non-classic Multinomial Models*

Haizhen Wu, PhD Student  
Victoria University of Wellington

The (classical) multinomial models, where the number and probabilities of the disjoint events are fixed, have been applied to many statistical problems such as goodness-of-fit tests, categorical analysis and diversity analysis. The limit distributions of many divisible statistics, such as Pearson's Chi-square statistic and log-likelihood ratio statistic, in classical models have been well-established.

However, many interesting statistical problems require a non-classical multinomial models, where the number of events increases and the probabilities change as the sample size increases. The asymptotic behaviour of the divisible statistics in these models, has not yet been thoroughly explored. In fact, even for the most classical Pearson's Chi-square statistic, its asymptotical behavior is still not clear in some non-classical multinomial models.

Although many recent papers still focus on the limit distributions of the divisible statistics, an advanced martingale approach, which instead study the functional limits of the partial sum processes of the divisible statistics, has been developed in 1980s by Khamladze. We extended this approach to more general situation and achieved similar functional limit theorems.

In this talk, we will focus on the properties of the partial sum processes, in particular the behaviour of their limiting process, to show the advantage of the martingale approach as well as the some applications of the functional limit theorems.

*User equilibria in networks: probabilistic routing vs. state-dependent routing*

Ilze Ziedins

The University of Auckland

Many networks, such as transportation networks, allow individuals to choose routes through the system to minimize their own delay, without considering the effect on others. However, system performance in such networks may be far from optimal, and adding capacity to the network may even degrade performance, rather than improving it. I will give some examples of this, and then examine a system where users are given additional information about the state of the system when making their routing choice. Questions of interest here are – do equilibrium policies exist, are they unique, and are delays reduced by giving users additional information about the state of the system. This talk draws on joint work with Heti Afimeimounga, Bruce Calvert and Wiremu Solomon.

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The Programme Committee,

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