Linear Programming and Critical Path Analysis

Peter McClymont - WGC
Rachel Hunt - VUW

3.2 Linear Programming

Secondary School (WGC)

- Internally assessed standard
- At WGC it’s only done in the Stats B (internal) course.
- 12 Level 2 M&S credits needed to do this course
- This standard requires no technology beyond a graphics calculator for finding the intersection of lines

Secondary School (WGC)

- Material covered:
  - linear inequalities
  - feasible regions
  - optimisation

Achievement	Achievement with Merit	Achievement with Excellence
Apply linear programming methods in solving problems.	Apply linear programming methods, using relational thinking, in solving problems.	Apply linear programming methods, using extended abstract thinking, in solving problems.

Example

Kiwi Klams is an exporter selling locally sourced clams overseas.
They export trays of large clams and trays of small clams.

This activity requires you to use linear programming to:

- model the constraints that Kiwi Klams have for the number of trays of large and small clams they can export
- make recommendations about the optimum number of trays they should export in order to maximise their profits
- discuss how changing their airline will affect profits.

You will present your findings supported by graphs, equations and relevant calculations.

Example

Kiwi Klams' contract with the supplier specifies that they will buy at least 20 trays of small clams, and at least 15 trays of large clams per week. The small clam constraint can be written as $S \geq 20$.

To purchase the clams from the supplier each tray costs the exporter $5 for small clams and $10 for large clams. Kiwi Klams have $800 available to spend on buying clams each week.

The contract with the airline is for for up to 100 trays per week.

After the airline and other costs are taken into account, the profit per tray is $15 for small clams and $20 for large. So Kiwi Klams' profit is given by the equation $P = 15S + 20L$, where S is the number of trays of small clams and L is the number of trays of large clams.

The exporter is considering changing the airline they use to get the clams overseas. If they do, then the profit per tray will be $12 for small clams and $24 for large.

Example

Constraints:

S ≥ 20 (given)

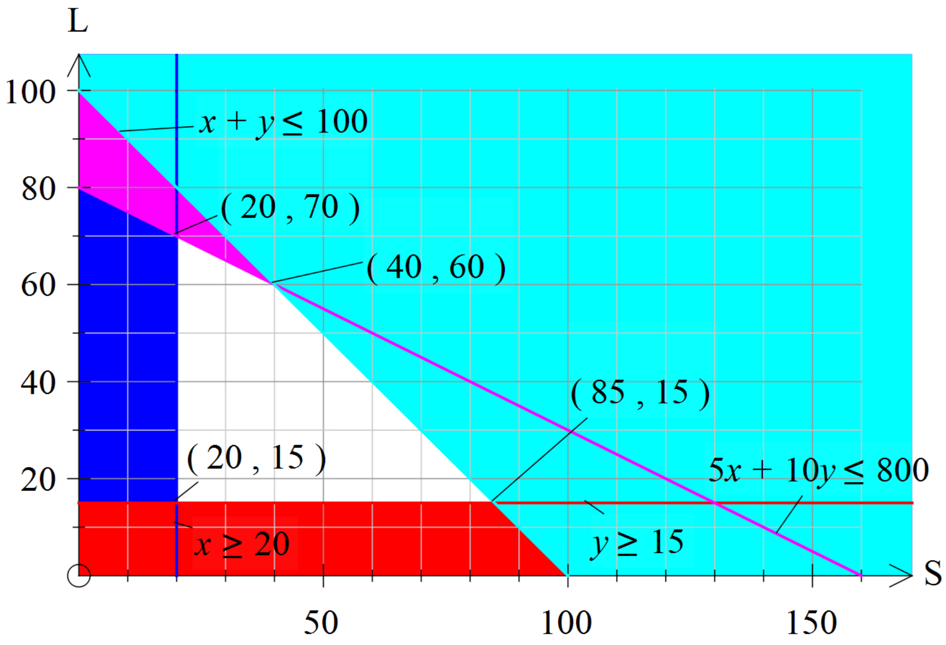
L ≥ 15

5S + 10L ≤ 800

S + L ≤ 100

Achieved requires:

- two correct contraints
- three consistent constraints plotted
- three consistent vertices
- identifying which vertex gives maximum consistent profit



Example

Merit requires:

- all contraints correct
- all constraints plotted
- profit calculated at appropriate vertices
- correct answer given incontext

(Small, Large)	Profit: P = 15S + 20L
(20,15)	$600
(20,70)	$1700
(40,60)	$1800
(85,15)	$1575

Kiwi Klams should export 40 trays of small clams, and 60 trays of large clams in order to maximise their profits, which will be $1800.

Example

(Small, Large)	New Profit: P = 12S + 24L
(20,15)	$600
(20,70)	$1920
(40,60)	$1920
(85,15)	$1380

Excellence requires merit plus:

- new profit correctly calculated in context
- whole number values
- objective function and constraint parallel

Kiwi Klams should change airlines, as they can make a profit of $1920 per week, an increase of $120.

They can do this if they export 40 trays of small clams and 60 trays of large clams as they did before, or 20 trays of small clams, and 70 trays of large clams, or any combination of whole number values on the line 5S + 10L = 800 between (20,70) and (40,60), e.g. 22 small and 69 large, or 24 small and 68 large, or 30 small and 65 large.

This is because the new profit equation (P = 12S + 24L) has the same gradient as the cost constraint (5S + 10L ≤ 800), and this results in multiple optimal solutions.

Victoria University

COURSE	Makes use of 3.2?	Assumes knowledge from 3.2?	Any issues without knowledge?
ENGR121	No	No	No
ENGR122	No	No	No
MATH132	No	No	No
MATH141	No	No	No
MATH142	No	No	No
MATH151	No	No	No
MATH161	No	No	No
MATH177	No	No	No
STAT193	No	No	No

Is this material ever used??

- Not until 300 level
- **MATH353**

Other Universities?

- The University of Auckland:
  - ENGSCI 255 - Modelling in Operations Research (15 points)
    "Emphasises the relationship between business and industrial applications and their associated operations research models. Software packages will be used to solve practical problems. Topics such as: **linear programming**, transportation and assignment models, network algorithms, queues, inventory models and simulation will be considered."
  - ENGSCI 391 - Optimisation in Operations Research (15 points)
  - ENGSCI 313 - Mathematical Modelling 3ECE (15 points)
- The University of Otago:
  - MATH151 - General Mathematics
    "In particular you will cover such topics as linear and quadratic models, **linear programming**, functional notation, differentiation, rates of change, graphing of functions, optimization problems, exponentials and logarithms, compound interest, exponential growth and decay, simple integration."

Is it useful to teach 3.2 then?

- Yes!
- Creating interest in, and showing that mathematical techniques are useful in real-world scenarios

3.4 Critical Path Analysis

Secondary School (WGC)

- Internally assessed standard
- At WGC it's only done in the Stats B (internal) course.
- 12 Level 2 M&S credits needed to do this course
- This standard requires no technology, but students can use a calculator if required.

Secondary School (WGC)

- Material covered:
  - precedence tables
  - network diagrams
  - critical events
  - scheduling
  - float times

Achievement	Achievement with Merit	Achievement with Excellence
Use critical path analysis in solving problems.	Use critical path analysis, with relational thinking, in solving problems.	Use critical path analysis, with extended abstract thinking, in solving problems.

Example

A company has been contracted to manage a project to build a new shopping mall.

Prepare a report recommending how the company should allocate staff to supervise its building. The report should include:

- a network diagram showing the tasks in the project
- the tasks in the critical path of the project
- the minimum length of time required to complete the project
- the start and finish time for each task
- the allocation of tasks to a minimum number of supervisors, given that each supervisor will only supervise one task at a time
- a recommendation of a week when the manager can visit the site and see at least three different tasks in action
- discussion about impact on the project if Task D takes longer than 3 weeks to complete
- discussion about impact of any other delays significant enough to extend the project's

Example

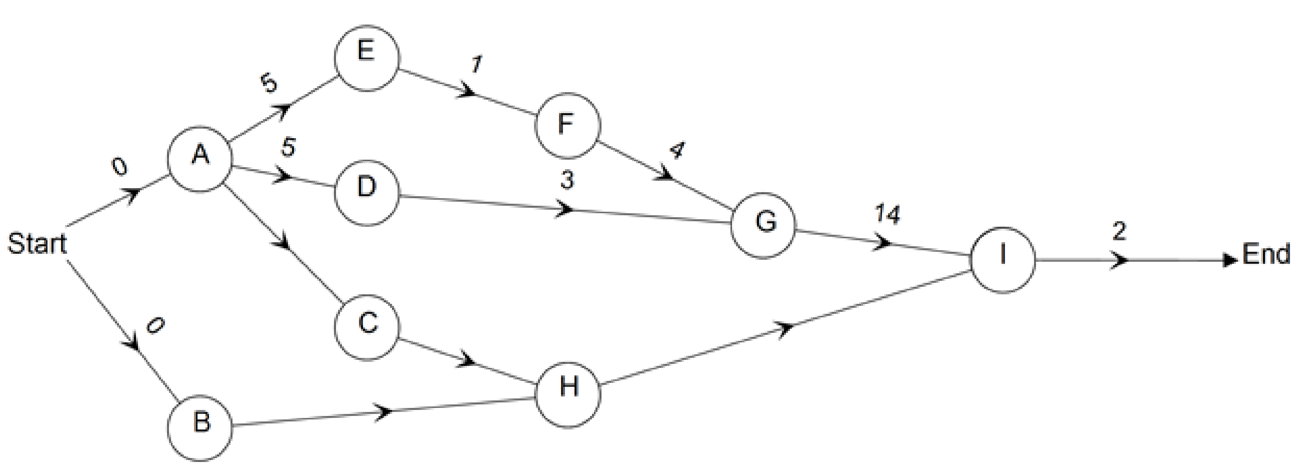
Senior staff will be allocated to supervise these tasks. Each person can only supervise one task at a time.

The company is aware that selection of the construction company may take longer than the time allowed in the table due to the competitive tendering process involved.

The manager is aware that factors outside their control could lead to delays in the completion of any task.

Task	Task name	Duration (weeks)	Preceded by
A	Prepare drawings	5	–
B	Identify tenants	6	–
C	Develop prospectus	4	A
D	Select construction company	3	A
E	Prepare resource consents	1	A
F	Obtain resource consents	4	E
G	Build mall	14	D, F
H	Finalise contracts with tenants	12	B, C
I	Tenants move in	2	G, H

Example



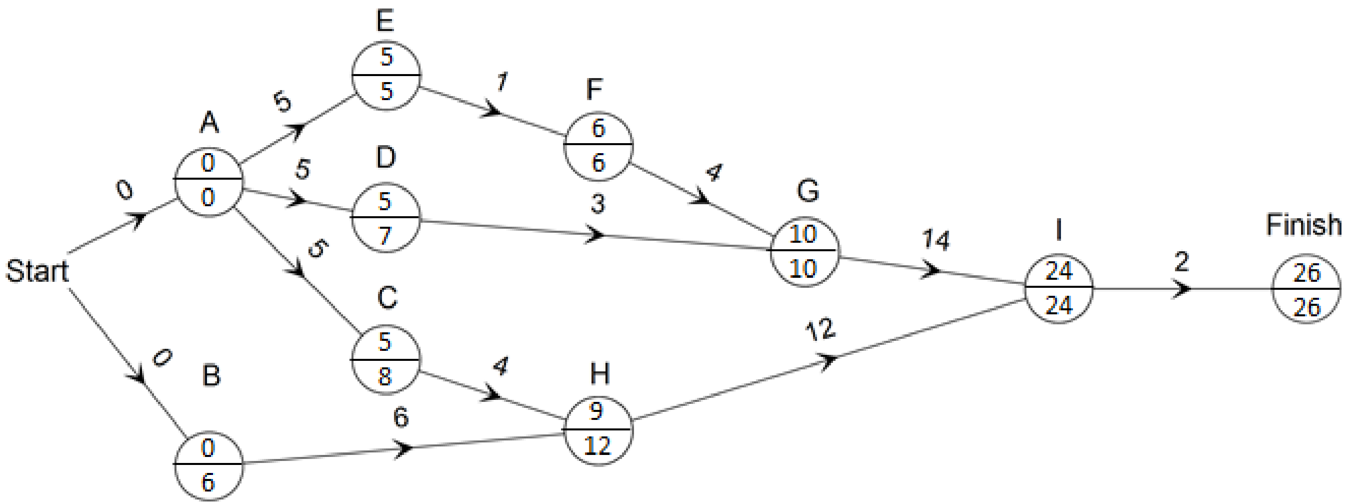
Critical Path: Start, A, E, F, G, I, End

It will take 26 weeks

Achieved:

- draw a network diagram
- find the critical path
- establish minimum time for the project

Example



Merit:

- establish earliest and latest start times using forward/backward scans
- allocate workers to tasks
- recommend appropriate review point to check progress

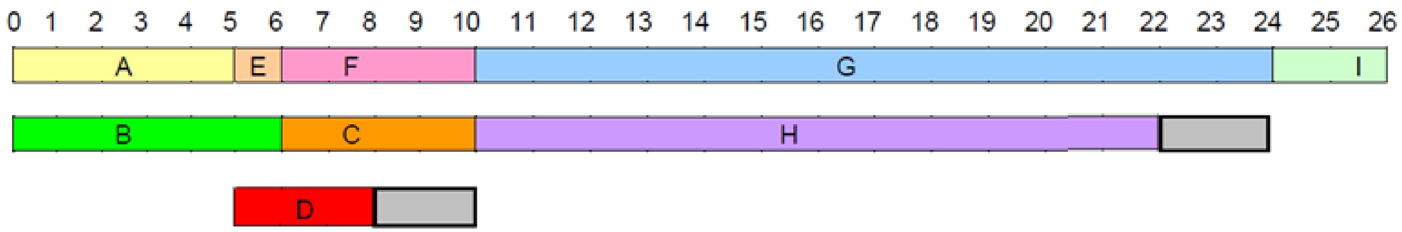
Example

Task	Earliest Start Time	Latest Start Time
A	0	0
B	0	6
C	5	8
D	5	7
E	5	5
F	6	6
G	10	10
H	9	12
I	24	24

Merit:

- establish earliest and latest start times using forward/backward scans
- allocate workers to tasks
- recommend appropriate review point to check progress

Example



For this allocation of tasks, I recommend that the manager visit at 5 weeks.

If task D takes longer than 3 weeks it may extend the project, especially if it finishes after week 10. For example, if it starts after 5 weeks and takes 6 weeks to complete, then the project will take 27 weeks in total.

Merit:

- establish earliest and latest start times using forward/backward scans
- allocate workers to tasks
- recommend appropriate review point to check progress

Example

Task	EST	LST	Float T
A	0	0	0
B	0	6	6
C	5	8	3
D	5	7	2
E	5	5	0
F	6	6	0
G	10	10	0
H	9	12	3
I	24	24	0

Excellence:

- Determine float times
- Investigate impact of delays on critical path on other tasks
- Discuss the possibility of tasks taking longer than the allocated time to complete and also the impact of delays to non-critical tasks on the critical path, minimum time, start/finish times, and placement of review points

Example

Any delay to a task on the critical path will cause a corresponding delay to the time of the whole project. For example, if Task F takes 5 weeks instead of 4, then the project will take 27 weeks. Any delay in the B – C – H series of up to two weeks is acceptable, but if, for example, Task C takes 7 weeks, then Task H will be delayed by one week, causing Task I to be delayed by one week as well. In this case, the new critical path would be Start – A – E – C – H – I – Finish.

However, B and C can be done in any order, so if the start of B was severely delayed, then Task C could be done first, then B then H.

Excellence:

- Determine float times
- Investigate impact of delays on critical path on other tasks
- Discuss the possibility of tasks taking longer than the allocated time to complete and also the impact of delays to non-critical tasks on the critical path, minimum time, start/finish times, and placement of review points

Victoria University

COURSE	Makes use of 3.4?	Assumes knowledge from 3.4?	Any issues without knowledge?
ENGR121	No	No	No
ENGR122	No	No	No
MATH132	No	No	No
MATH141	No	No	No
MATH142	No	No	No
MATH151	No	No	No
MATH161	No	No	No
MATH177	No	No	No
STAT193	No	No	No

Is this material ever used??

- Not within School of Mathematics, Statistics and Operations Research.
- Critical path analysis appears in 300 level papers in other subject areas:
  - BILD361/SARC361 – Project Management

  (School of Architecture and Design)
  - ENGR301 – Project Management

  (School of Engineering and Computer Science)
- Knowledge would **not** be assumed.

Linear Programming and Critical Path Analysis

Peter McClymont - WGC
Rachel Hunt - VUW