Secondary-Tertiary Transition in Mathematics and Statistics

ALGEBRA

(AS 3.5 and 3.15)

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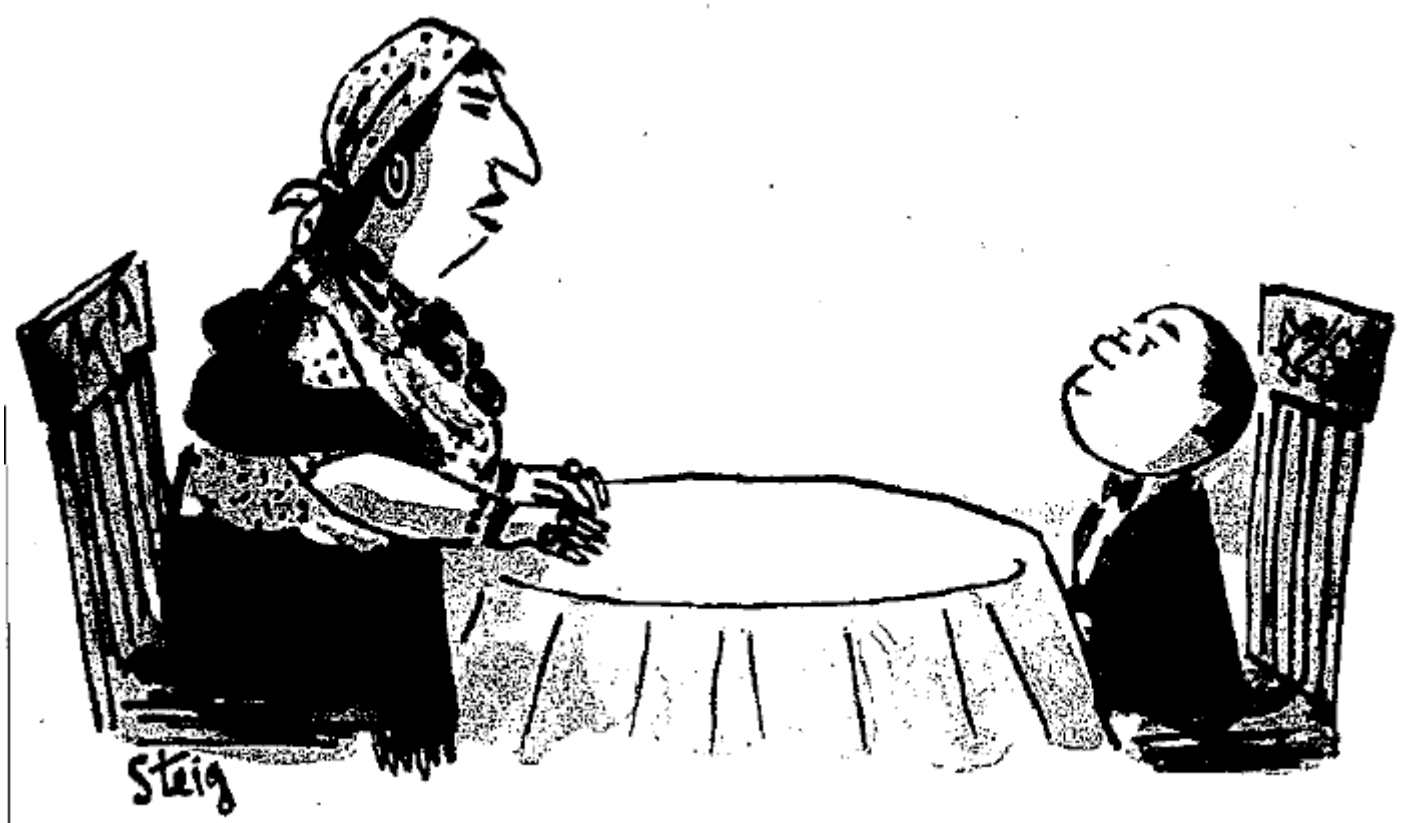
What algebra skills can tertiary educators expect students to arrive with?



"I see trouble with algebra."

Overview of Algebra Years 11 - 13

Level 1
AS 1.2 MCAT (4 credits “External”)
AS 1.3 Tables, Equations and Graphs (4 credits External)
AS 1.4 Apply linear algebra (3 credits - Int)

Level 2
AS 2.6 External (4 credits)
AS 2.14 Simultaneous Equations (2 credits - Int)
AS 2.1 Co-ordinate Geometry (2 credits - Int)
AS 2.2 Non-linear graphs (4 credits - Int)

Question: How much of this might a student who does a Level 3 Statistics course alone have done?

Level 3
AS 3.5 Complex Numbers (5 credits External)
AS 3.15 Simultaneous Equations (3 credits - Int)
(Algebra skills involved in Level 3 Trigonometry and Calculus standards)

Overview of Algebra Years 11 - 13

Level 2 – AS 2.6 External (4 credits)

Methods include a selection from those related to:

- manipulating algebraic expressions, including rational expressions
- manipulating expressions with exponents, including fractional and negative exponents
- determining the nature of the roots of a quadratic equation
- solving exponential equations (which may include manipulating logarithms)
- forming and solving linear and quadratic equations.

2.6 paper 2013

Algebra in Level 3 NCEA

AOs from Level 8 of *The New Zealand Curriculum*:

- Manipulate complex numbers and present them graphically
- Form and use polynomial, and other non-linear equations
- Form and use systems of simultaneous equations, including three linear equations and three variables, and interpret the solutions in context

AS 3.5 Apply the algebra of complex numbers in solving problems

Methods are selected from those related to:

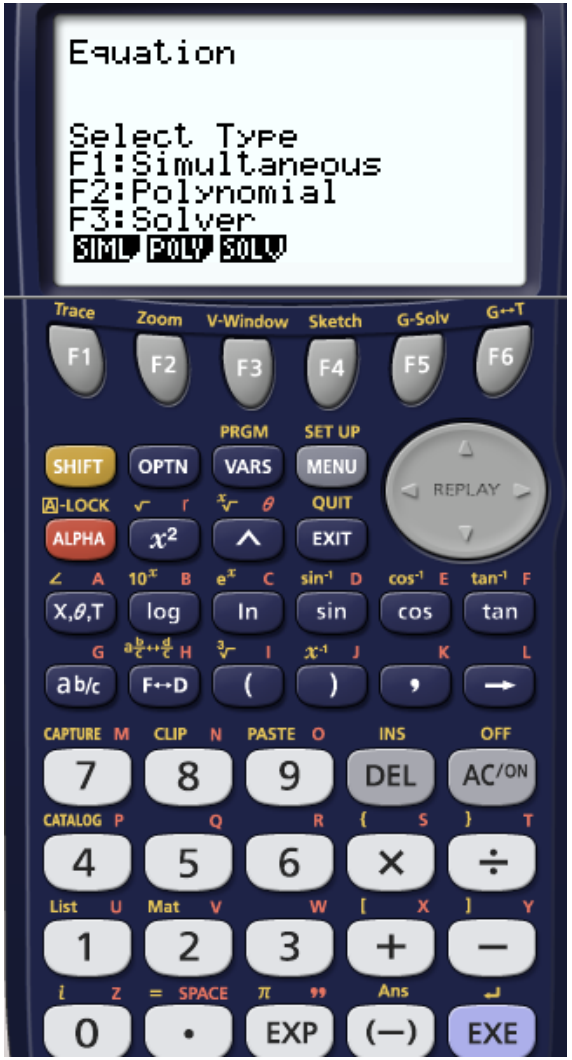
- quadratic and cubic equations with complex roots
- Argand diagrams
- polar and rectangular forms
- manipulation of surds
- manipulation of complex numbers
- loci
- De Moivre's theorem
- equations of the form $z^n = r\,\mathrm{cis}\,\theta$, or $z^n = a + b\,\mathrm{i}$ where a, b are real and n is a positive integer.

2013 NZQA paper

Impact of technology – graphics calculators

AS 3.5 Complex Numbers

- manipulation of complex numbers
- polynomial equations
- surd equations



Solve the following equation for x in terms of p:

$$\sqrt{x}-3=\sqrt{x-p}$$

$$x-6\sqrt{x}+9=x-p$$

$$6\sqrt{x}=p+9 \qquad (3)$$

$$36x=(p+9)^2$$

$$x=\frac{(p+9)^2}{36}$$

$$\text{or } x=\frac{p^2+18p+81}{36} \text{ or } x=\frac{p^2}{36}+\frac{p}{2}+\frac{9}{4}$$

Question: How much of a paper would a student have to complete to get an Achieved grade?

AS 3.15 Apply systems of simultaneous equations in solving problems

Methods include a selection from those related to:

- forming systems of simultaneous equations
- solving systems of simultaneous equations
- the nature of solutions to systems.

Clarifications:

- To be used as evidence for the award of Achieved a ‘method’ must be relevant to the solution of the problem and at the appropriate curriculum level.
- At all levels there is a requirement relating to the communication of the solutions.

AS 3.15 NZQA approved exemplar task

Task

Elaine sees some equations in a book and they remind her of systems of equations she has used in her mathematics class. The first two equations are:

$2x + 2z = 3y + 1$

$y = 4z + 8$

Elaine needs a third equation and decides to use three different methods to create it. Use each of these methods to create the third equation:

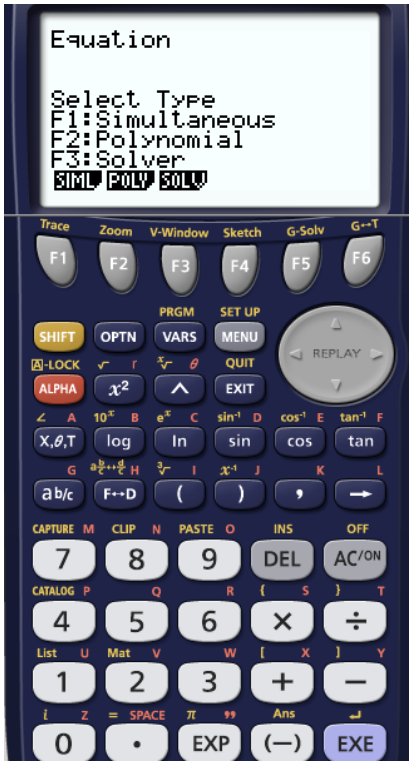
- use her pin number, 3287, as the coefficients of *x*, *y*, and *z*, and the constant term in that order when the equation is in the form ax + by +cz = d
- multiply all coefficients and the constant in the first equation, 2x + 2z = 3y + 1, by 3
- use the first equation, 2x + 2z = 3y + 1, but change the constant from 1 to 6.

Solve each set of three equations and give a geometric interpretation of the solution. Write a general statement about the solution for each set.

As you write your report, include the equations you have used, as well as relevant calculations and/or diagrams. Use appropriate mathematical statements to communicate your findings.

Impact of technology – graphics calculators

AS 3.15 Simultaneous Equations

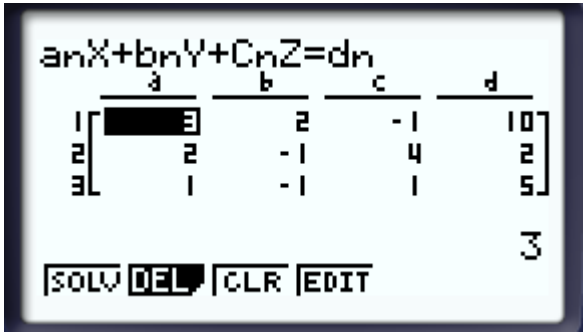


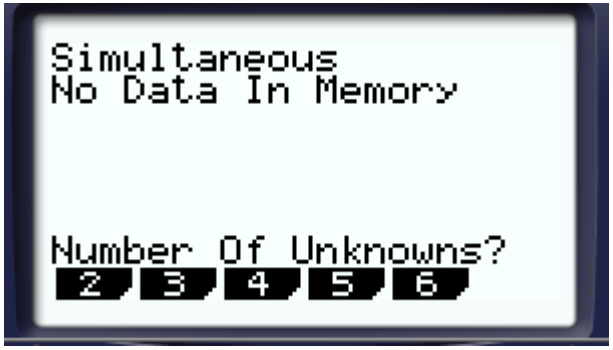
Solve the set of equations

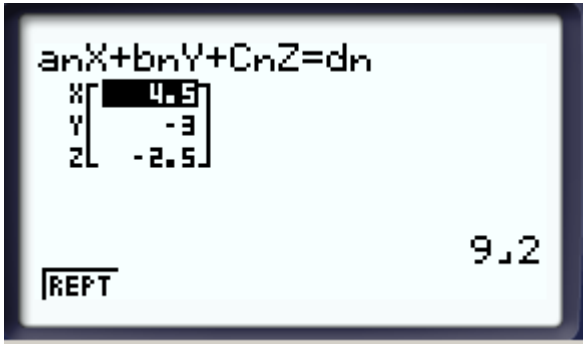
$3x + 2y - z = 10$

$2x - y + 4z = 2$

$x - y + z = 5$







Answer: x = 4.5, y = -3, z = -2.5

Who does 3.5 and 3.15?

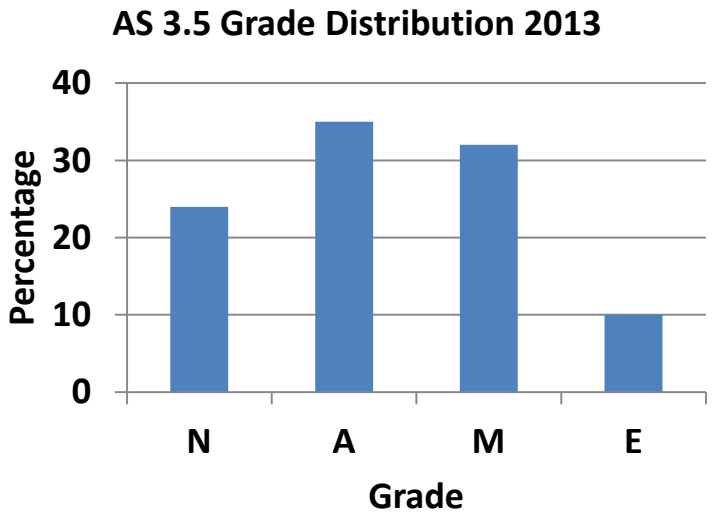
(Data from 2013, the first year of the realigned Level 3 standards)

AS 3.5

7 564 students

(c.f. 8722 and 7862 for the Differentiation and Integration external standards respectively)

Students doing a Calculus course

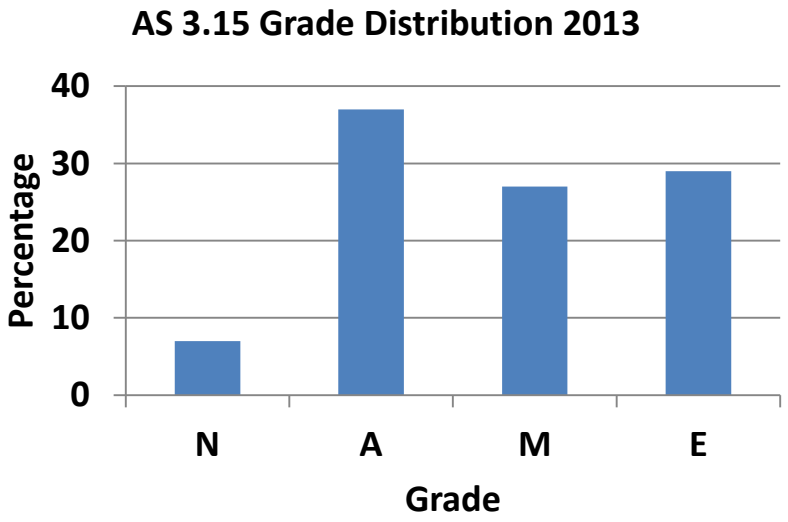


AS 3.15

12 453 students

Students doing

- a Calculus course
- a Statistics course
- a Mathematics (and Statistics) course
- Year 12 Maths?



VUW courses

- Topics in 3.5 and 3.15 most closely correspond to content in the following VUW courses :
  - MATH151 Algebra
    - (matrices and vectors, complex numbers, eigenvectors, algebraic structures)
  - ENGR122 Engineering Mathematics with Calculus
    - (calculus, differential equations, linear algebra)

Comparison of sample questions

- MATH151
  - Find the real and imaginary parts of $(4+2i)(3-5i)$.
  - Find the Cartesian form of the number $(1-i)^8$.
- NCEA 3.5
  - If $u=3-3i$, find u^4 in the form $r\text{cis}\theta$.
  - Given $u=3+2i$, find the Cartesian form of $\bar{u}^2+1/u^2$.
- ENGR122
  - Find the real and imaginary parts of $3e^{i\pi/4}$.

Comparison of sample questions

- MATH151
  - ax^3+bx^2+3x has remainders -5 on division by x-1, and 10 on division by x-2. Find a and b.
  - Find the roots of $x^2-4x+4-i$.
- NCEA 3.5
  - Given x-2 is a factor of x^3-2px^2+px-5, find the value of p.
  - Solve the equation $x=\sqrt{(33-4x)} + 3$

Comparison of sample questions

- NCEA 3.15
  - Roger wants his rabbits' daily vitamin intake to be:
    - 1000 micrograms (μg) of vitamin A
    - 1600 milligrams (mg) of vitamin C
    - 2400 milligrams (mg) of vitamin E.
  - Each gram of Xena contains 2 μg of vitamin A, 3 mg of vitamin C, and 5 mg of vitamin E.
  - Each gram of Yum contains 4 μg of vitamin A, 7 mg of vitamin C, and 9 mg of vitamin E.
  - Each gram of Zany contains 5 μg of vitamin A, 10 mg of vitamin C, and 14 mg of vitamin E.

Comparison of sample questions

- MATH151
  - Reduce each one of the matrices below to a matrix in reduced row echelon form…
  - Solve the following system of equations
    - $3x+2y-z=10$
    - $2x-y+4z=0$
    - $-x-y+z=1$

Differences in geometric approach

- NCEA 3.5: u and v are shown on an Argand diagram, indicate the location of u-v.
  - The exemplar solution finds the Cartesian coordinates of u and v, performs the subtraction, and then locates u-v on the diagram.
  - By contrast, MATH151 explicitly emphasises addition of complex numbers as being vector addition.

Differences in geometric approach

- Similarly, in 3.15, geometric reasoning about the system of equations is not necessary to achieve Excellence.
- Perhaps VUW can produce added-value by emphasising geometric approaches to complex numbers and simultaneous equation.