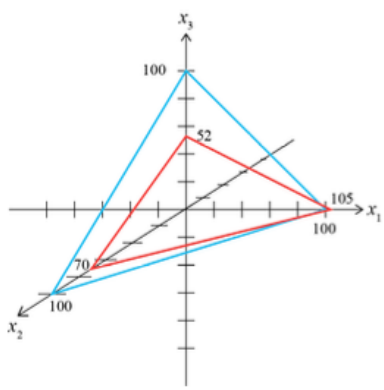
Question 3 solutions (2016 Q5)

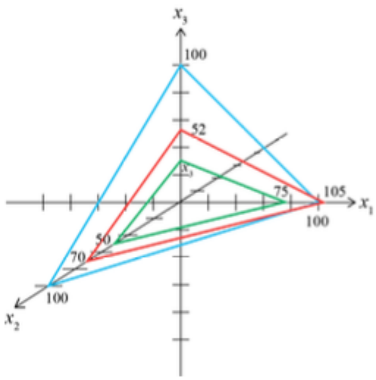
(b) ***EITHER***

	Question Type		
	Group 1 (x_1)	Group 2 (x_2)	Group 3 (x_3)
Marks	4	5	6
Time	2	3	4

$x_1 + x_2 + x_3 \leq 100$ constraint 1
$2x_1 + 3x_2 + 4x_3 \leq 210$ constraint 2
$2x_1 + 3x_2 \leq 150$ constraint 3
$x_1 \geq 0$ constraint 4
$x_2 \geq 0$ constraint 5
$x_3 \geq 0$ constraint 6
Objective function: Grade = $4x_1 + 5x_2 + 6x_3$
Feasible Solutions are found at vertices. (0,0,0) is feasible but not helpful. Constraint (1) has vertices (100,0,0), (0,100,0) and (0,0,100). However, constraints 3 and 2 define the max values for the question groups as $x_1 \leq 75$, $x_2 \leq 50$ and $x_3 \leq 52$.
Constraint 2 has vertices (105,0,0), (0,70,0), (0,0,52.5). This plane lies mostly between the plane defined by constraint 1 and the origin. All intersections lie in a region where $x_1 > 75$. By constraint 3, no feasible solutions in this region. Any feasible solutions now lie between plane 2 and the origin.
Only one vertex of plane 2 offers a feasible solution, being (0,0,52.5). After truncation, the objective function yields Grade = $4 \times 0 + 5 \times 0 + 6 \times 52 = 312$.
The vertices of constraint 3 are (75,0,0), (0,50,0) and (0,0,x_3): a plane with one side fixed and the other two dependent on x_3.



This plane lies between the planes defined by constraints 1 & 2 and the origin, with no intersections.
Objective function applied to the vertices (75,0,0) and (0,50,0) gives us, respectively,
Grade = $4 \times 75 = 300$ and Grade = $5 \times 50 = 250$. We have no improvement on 312.
The value of x_3 in constraint 3 is "checked" by constraint 2. Consider the boundary equations from constraints 2 & 3:
$2x_1 + 3x_2 + 4x_3 \leq 210$ eq 2
$2x_1 + 3x_2 = 150$ eq 3
Eq 2 – eq 3 gives $4x_3 = 60$ or $x_3 = 15$.
So, fixing $x_3 = 15$ means the other two vertices of constraint 3 are (75,0,15) and (0,50,15).



Applying these vertices to the objective function gives:

(0,0,15) Grade = $6 \times 15 = 90$

(75,0,15) Grade = $75 \times 4 + 15 \times 6 = 390$

(0,50,15) Grade = $50 \times 5 + 15 \times 6 = 340$

The student should therefore complete 75 group 1 questions and 15 group 3.