CURRENT TRENDS IN RANDOM MATRIX ANALYSIS

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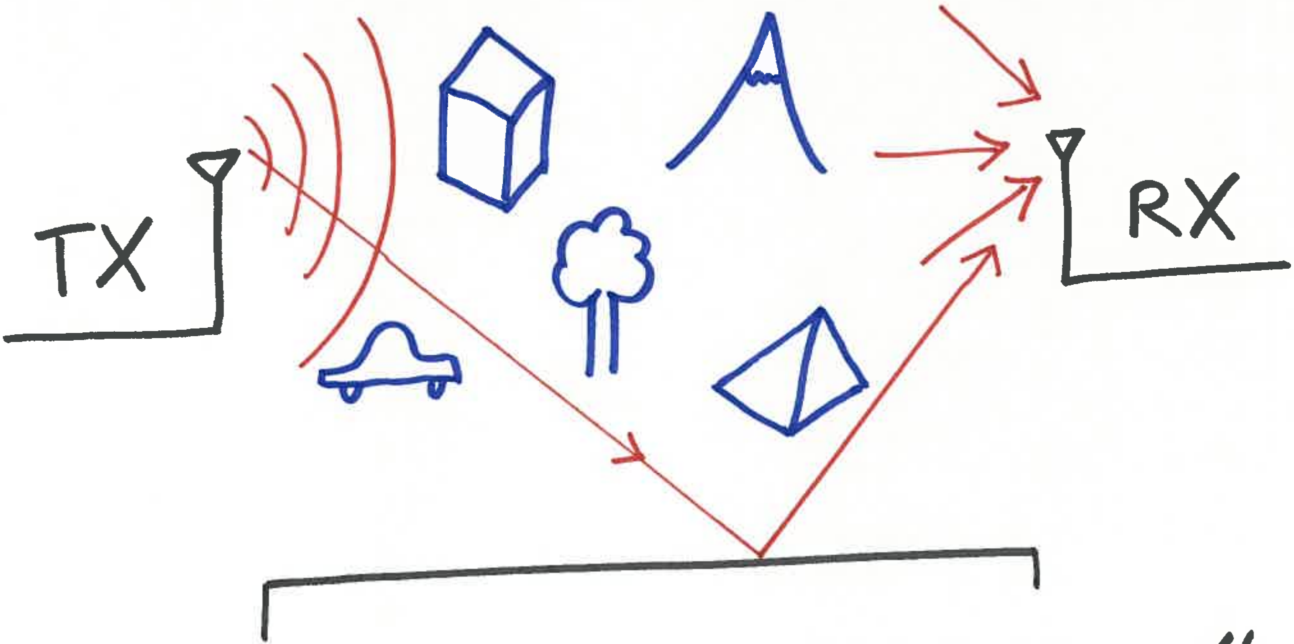
MICROWAVE → mmWAVE

CLASSICAL M.V. STATS



channel = fixed dominant path
+ many independent paths

$$= \mu + \mathcal{CN}(0, \sigma^2)$$

Complex normal channels

Channel Matrix

$$H = H_{LOS} + H_{SC}$$

CONST. $\qquad$ $\mathbb{C}N(0,?)$

$$C = \log_2 \left[\left| I + \frac{\rho}{N} H H^{\dagger} \right| \right]$$

$$= \sum \log_2 \left(1 + \frac{\rho}{N} \lambda_i \right)$$

NEED EIGENVALUES OF QUADRATIC FORMS IN $\mathbb{C}N$!

James
Muirhead
Khatri
et al...

THE METHODOLOGY

$$f(\underset{\sim}{\lambda}) = K \prod_{i=1}^{m} g(\lambda_i)\, V(\underset{\sim}{\lambda})\, \Delta$$

$$\Delta = \left| \begin{bmatrix} \vdots & \vdots & & \vdots \\ c(\lambda_1) & c(\lambda_2) & \cdots & c(\lambda_m) \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \end{bmatrix} \right|$$

$$V(\underset{\sim}{\lambda}) = \begin{vmatrix} 1 & \lambda_1 & \cdots & \lambda_1^{m-1} \\ 1 & \lambda_2 & \cdots & \lambda_2^{m-1} \\ \vdots & \vdots & & \vdots \\ 1 & \lambda_m & \cdots & \lambda_m^{m-1} \end{vmatrix}$$

TRICK

$$V(\underset{\sim}{\lambda}) = \sum_{\alpha} (-1)^{\mathrm{per}(\alpha)} \lambda_1^{\alpha_1} \lambda_2^{\alpha_2} \cdots \lambda_m^{\alpha_m}$$

OUTCOME

$$f(\underset{\sim}{\lambda}) = K \prod_{i=1}^{m} g(\lambda_i) \sum_{\alpha} (-1)^{per(\alpha)} \prod_{i=1}^{m} \lambda_i^{\alpha_i} \Delta$$

$$= K \sum_{\alpha} (-1)^{per(\alpha)} \Delta_2$$

$$\Delta_2 = \begin{bmatrix} g(\lambda_1)\lambda_1^{\alpha_1} c(\lambda_1) & \cdots & g(\lambda_m)\lambda_m^{\alpha_m} c(\lambda_m) \\ \vdots & & \vdots \\ \vdots & & \vdots \end{bmatrix}$$

"COLUMN SEPARABLE"

IMPLICATION

In principle any analysis which is truly separable (in both eigenvalues and in range) has a mechanism for solution.

EXAMPLE

SPECTRUM SENSING....

.... or the "sphericity test."

<u>Require</u>

$$T = E\left[\log\det\left(R^{1/2}HH^{\dagger}R^{1/2}\right)\right]$$

$$H \sim \mathbb{C}N(0, I)$$

$$\boxed{T = \log\det(R) + E\sum_{i=1}^{m}\log\lambda_i}$$

ie., we need $E[\log\lambda_1]$ where λ_1 is an arbitrary eigenvalue.

$$E\left[\log \lambda_1\right]$$

$$= \int\int\cdots\int \log \lambda_1 \; \frac{1}{m!} f(\underset{\sim}{\lambda}) \; d\underset{\sim}{\lambda}$$

$$= \frac{K}{m!} \sum_{\alpha} (-1)^{per(\alpha)} \underbrace{\int_0^\infty \cdots \int_0^\infty \log \lambda_1 \times \Delta_2 \; d\underset{\sim}{\lambda}}_{I_\alpha}$$

$$I_\alpha = \begin{bmatrix} \int_0^\infty \log \lambda_1 \, g(\lambda_1) \lambda_1^{\alpha_1} c(\lambda_1) d\lambda_1 & \cdots & \int_0^\infty g(\lambda_m) \lambda_m^{\alpha_m} c(\lambda_m) d\lambda_m \\ \vdots & & \vdots \end{bmatrix}$$

This route has solved many comms problems in the last 15 years.

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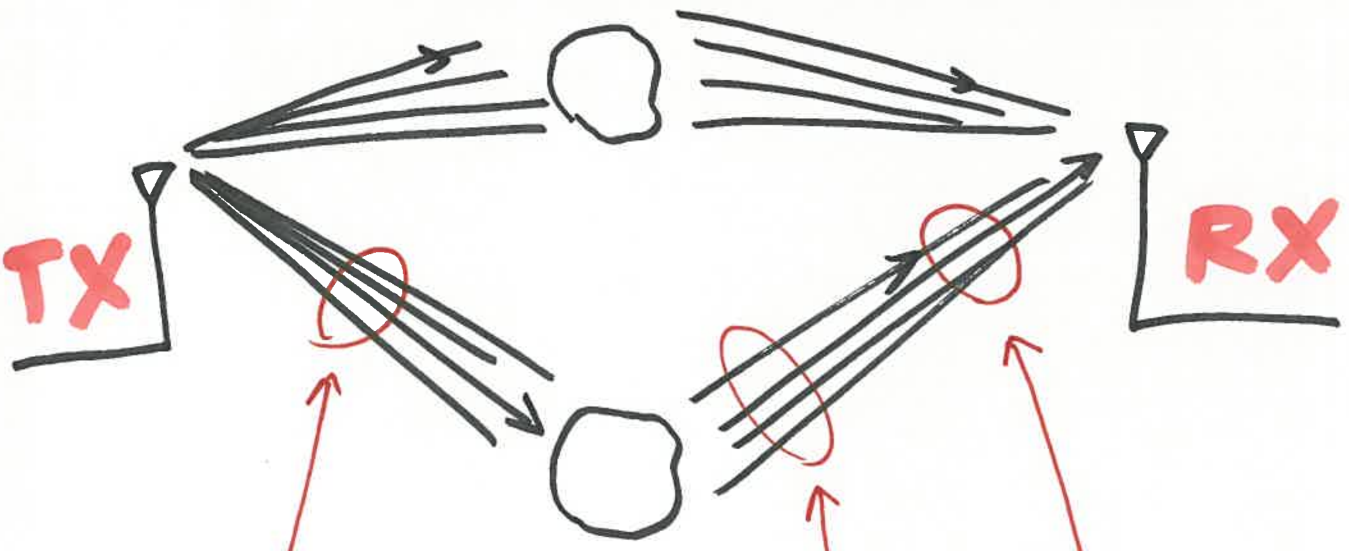
mmWave Analysis

mmWAVE CHANNELS



random sub angles around a random cluster angle

AoD

AoA

Random powers

CLUSTERS (1-4)

Goodbye CN !

$$H = \sum_{i=1}^{C} \sum_{j=1}^{L} g_i \, \underset{\sim}{u}_r\left(\theta_{ij}^{IN}\right) \underset{\sim}{u}_t\left(\theta_{ij}^{OUT}\right)^{\dagger}$$

C clusters

L sub-paths

$|g_i|^2$ = random power for cluster i

$g_i \sim \mathbb{C}N(0, P_i)$

$$\underset{\sim}{u}_r(\theta) = \begin{bmatrix} 1 \\ e^{jk\cos(\theta)} \\ e^{2jk\cos(\theta)} \\ \vdots \\ e^{(M-1)jk\cos(\theta)} \end{bmatrix}$$

RANDOM POWERS

RANDOM ANGLES

Callum

WHAT IS KNOWN ABOUT H?

<u>A.</u> VIRTUALLY NOTHING!

Certainly no eigenvalue results

extreme – Himal

Progress

- FORGET EXACT RESULTS
- FOCUS ON METHODS WHICH ONLY REQUIRE MOMENTS

↓

example

Example

$$\left(HH^{\dagger}\right)_{rs} = \sum_{i=1}^{C}\sum_{j=1}^{L}\sum_{i'=1}^{C}\sum_{j'=1}^{L} g_i g_{i'}^{*} \, \underset{\sim}{u}_{r,r}(\theta_{ij}^{IN}) \underset{\sim}{u}_t(\theta_{ij}^{OUT})^{\dagger}$$
$$\times \, \underset{\sim}{u}_t(\theta_{i'j'}^{OUT}) \, u_{r,s}^{*}(\theta_{i'j'}^{IN})$$

$$E\left(HH^{\dagger}\right)_{rs} = \sum_{i=1}^{C}\sum_{j=1}^{L} E|g_i|^2 \times M \times E\left[\underset{\sim}{u}_{rr}(\theta_{ij}^{IN}) \, \underset{\sim}{u}_{rs}^{*}(\theta_{ij}^{IN})\right]$$

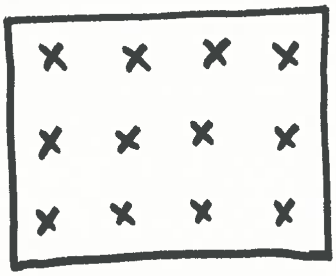
$$= M \sum_{i=1}^{C}\sum_{j=1}^{L} P_i \, E\left\{ e^{jk(r-1)\cos(\theta_{ij}^{IN})} \times e^{-jk(s-1)\cos(\theta_{ij}^{IN})} \right\}$$

Hence, the angular distribution of the incoming radiation is important:

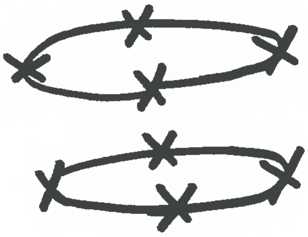
$$E\left[e^{jk(r-s)\cos\theta}\right]$$

- NOTE: Simplest case...

Rectangular arrays



Circular arrays



$$\left(\underset{\sim}{u}_r(\theta)\right)_k = e^{j(a_r \sin\theta \cos\phi + b_r \sin\theta \sin\phi + c_r \cos\theta)}$$

θ = elevation

ϕ = azimuth