There are no maximal dre wtt-degrees

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Jump Inversion Theorems:

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- Low degrees and High degrees, High/Low hierarchy
- Sacks: There exist intermediate c.e. degrees, i.e. not low_n, not high_n for any n.

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- We cannot strengthen "Turing reduction" above as "*wtt*-reduction", as Bickford and Mills also proved
  - If A is superlow and $\emptyset'$ is *wtt*-reducible to $A \oplus W$, then $\emptyset'$ is *wtt*-reducible to W.

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  - If A is superlow and $\emptyset'$ is *wtt*-reducible to $A \oplus W$, then $\emptyset'$ is *wtt*-reducible to W.
- This shows the existence of c.e. sets, low, but not superlow.

Bounded Jump Operator - a definition of Anderson and Csima

For $A \subseteq \mathbb{N}$, define

$$A^{\dagger} = \{x : \exists i < x[\varphi_i(x) \downarrow \ \& \ \Phi_x^{A \upharpoonright \varphi_i(x)}(x) \downarrow]\}.$$

Obviously, $A^{\dagger} \leq_T A \oplus \emptyset'$, so if $A \geq_T \emptyset'$, then $A^{\dagger} \equiv_T A$.

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We can also have:

- $\emptyset^\dagger \equiv_1 \emptyset'$. So, for set A, $A \leq_{wtt} \emptyset^\dagger$ if and only if A is ω-c.e.
- $A \leq_1 A^\dagger$ and
- $A^\dagger \not\leq_{wtt} A$.
- $A^\dagger \leq_1 A'$.
- For some set A, $A^\dagger \not\leq_{wtt} A \oplus \emptyset'$.

  As indicated above, $A^\dagger \leq_T A \oplus \emptyset'$ is always true.
- For sets A, B with $A \leq_{wtt} B$, $A^\dagger \leq_1 B^\dagger$.

  This shows that $\dagger$, as a jump, is well-defined.

An analogue of Shoenfield's Jump Inversion

Theorem (Anderson and Csima):

For a set C with $\emptyset^\dagger \leq_{wtt} C \leq_{wtt} \emptyset^{\dagger\dagger}$, there is a set $B \leq_{wtt} \emptyset^\dagger$ such that $C \equiv_{wtt} B^\dagger$.

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A set A is bounded-low if $A^\dagger \leq_{wtt} \emptyset^\dagger$, i.e. if $A^\dagger$ is ω-c.e..

1. All superlow sets are bounded-low.
2. Bounded-low sets can have high degree. (Anderson, Csima and Lange)
3. There is a superhigh bounded-low set. (G. Wu and H. Wu))
4. There is a low, but not superlow, bounded-low set. (G. Wu and H. Wu)

The structure of r.e. *wtt*-degrees

1. A splitting theorem of Ladner and Sasso, in contrast to Lachlan's nonsplitting theorem
2. Why does it work for *wtt*-degrees?
3. Distributivity, by Lachlan

Contiguous degrees

1. Definition and characterization of contiguous degrees
2. Contiguous degrees are low_2
3. Downey's strongly contiguous degrees and noncuppability
4. Stob's result of a contiguous degree as a top of minimal pair, and discontinuity
5. dre contiguous degree, and others (joint with Yamaleev)

No maximal dre *wtt*-degrees

1. CHLLS's maximal dre Turing degrees
2. Maximal dre Turing degrees cannot be low, by ACL and DY.
3. There are no maximal dre *wtt*-degrees (joint with Yamaleev)
4. Requirements and strategies

Thanks!