Observer dependent temperature of perceived radiation in black hole physics

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Outline

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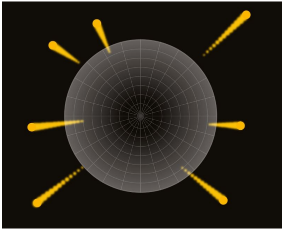
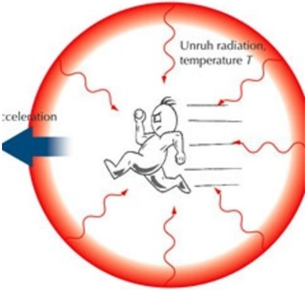
What is Hawking radiation?



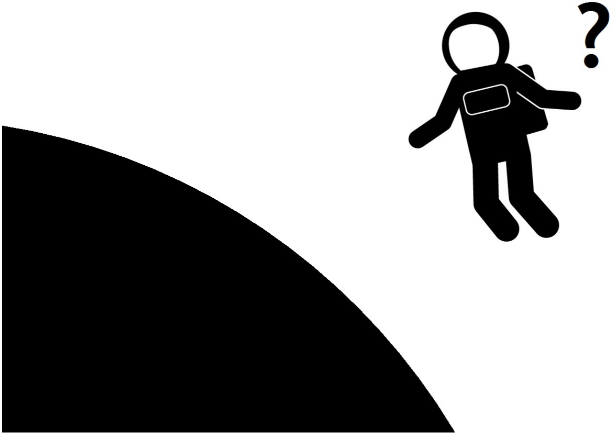
- It is a radiation of quantum nature that escapes from black holes
- It has its origin in the collapse process that forms the black hole
- For late enough times, it does not depend on its details
- The spectrum of the radiation is thermal, with temperature $T_{\rm H} = \hbar\kappa_{\rm H}/(2\pi k_{\rm B})$ proportional to the surface gravity $\kappa_{\rm H}$

What is the Unruh effect?

- Particle definition and perception in quantum field theory is an observer dependent notion
- In general, families of observers will differ in their notions of vacuum state and particles
- The Unruh effect: the thermal spectrum that an accelerated observer detects with temperature $T_{\mathrm{U}} = \hbar a/(2\pi k_{\mathrm{B}})$ proportional to its acceleration



Our question



How these two effects combine to give the **net perception for a general observer** outside a black hole?

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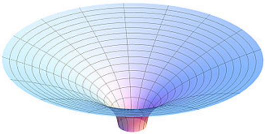
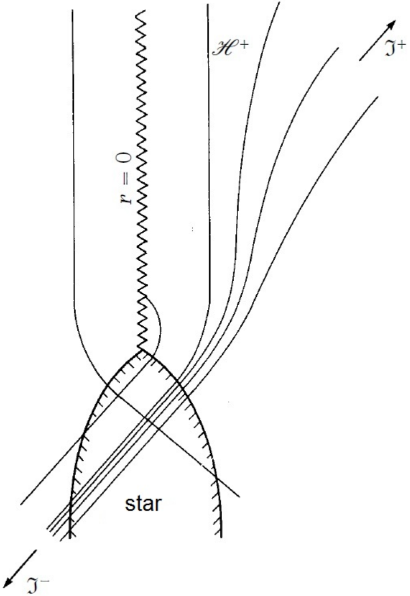
Geometry, field, and approximations



$$\Box\phi = 0$$

- Radial sector of Schwarzschild spacetime (we work in $1+1$ dimensions)
- We omit the *grey body factors*
- Massless real Klein-Gordon scalar field (conformal invariance)

Collapse scheme



Collapse process

- Ingoing normal modes in the asymptotic null past are $\mathrm{e}^{-\mathrm{i}\omega'\bar{v}}$, where $\bar{v} := t + r^*$ is the affine parameter in that region
- The field is on the state $|0_{\mathrm{in}}\rangle$, the vacuum state associated to these modes
- Outgoing normal modes in the asymptotic null future are $\mathrm{e}^{-\mathrm{i}\omega\bar{u}}$, where $\bar{u} := t - r^*$ is the affine parameter in that region
- The vacuum state associated to that modes would be $|0_{\mathrm{out}}\rangle$
- When reversed back through the dynamical collapse (from late enough times), $\mathrm{e}^{-\mathrm{i}\omega\bar{u}}$ modes end up in modes $\sim \mathrm{e}^{\mathrm{i}(\omega/\kappa_{\mathrm{H}})\log[A(\bar{v}_{\mathrm{H}}-\bar{v})]}$

Hawking radiation found

- These modes mix positive and negative frequency modes of the kind $\mathrm{e}^{-\mathrm{i}\omega' \bar{v}}$!
- If the vacuum is $|0_{\mathrm{in}}\rangle$, observers in the asymptotic null future will detect a non-zero amount of particles
- This amount happens to be

$$\langle N_\omega \rangle = \frac{1}{\mathrm{e}^{\frac{2\pi\omega}{\kappa_{\mathrm{H}}}} - 1}$$

- This is a thermal spectrum with temperature proportional to κ_{H}: Hawking radiation

The key point

- **The key point:** the transformation of the normal outgoing modes when reversed back in time
- Its properties are encoded in the relations between the affine parameters in the past ($\bar{v}$) and future ($\bar{u}$) regions

$$\bar{v} = \bar{v}_{\mathrm{H}} - \frac{1}{A} \mathrm{e}^{-\kappa_{\mathrm{H}} \bar{u}}$$

- A similar derivation in Minkowski vacuum for accelerated observers yields the Unruh effect

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Definition of the effective temperature function

- Consider the vacuum state $|0_U\rangle$ associated to some modes $\mathrm{e}^{-\mathrm{i}\omega' U}$
- Consider some observer $(t(\tau), r(\tau))$ who naturally couples to the modes $\mathrm{e}^{-\mathrm{i}\omega\tau}$
- Let us define the function

$$\kappa(\tau) := -\left.\frac{\mathrm{d}^2 U}{\mathrm{d}\tau^2}\middle/\frac{\mathrm{d}U}{\mathrm{d}\tau}\right.$$

- If $\kappa(\tau)$ is approximately constant $\approx \kappa_*$ around some instant τ^*, we integrate and obtain

$$U = U_{\mathrm{H}} - \frac{1}{A}\mathrm{e}^{-\kappa_* \tau}$$

- During this interval the observer perceives radiation with temperature $T_* = \hbar|\kappa_*|/(2\pi k_{\mathrm{B}})$

A general expression

- We have a general expression for $\kappa(\tau)$

$$\kappa(\tau) = \sqrt{\frac{1 - v_l}{1 + v_l}} \frac{1}{\sqrt{1 - \frac{2M}{r}}} \left(\bar{\kappa}(\bar{u}) - \frac{M}{r^2} \right) + a_p$$

- $r(\tau)$ is the radial position
- $v_l(\tau)$ is the local velocity with respect to the black hole
- $a_p(\tau)$ is the proper acceleration
- $\bar{u}(\tau)$ is the $\bar{u}$ parameter of the light ray it is crossing
- $\bar{\kappa}(\bar{u})$ is the effective temperature for static observers in the asymptotic region

A first attempt of interpretation

- $$\kappa(\tau) = \sqrt{\frac{1-v_l}{1+v_l}} \frac{1}{\sqrt{1-\frac{2M}{r}}} \left(\bar{\kappa}(\bar{u}) - \frac{M}{r^2} \right) + a_p$$
- $\bar{\kappa}(\bar{u})$ is the escaping radiation that the observer is crossing at τ
- $-\frac{M}{r^2}$ is a subtracting term due to ‘gravitational acceleration’ (but why?)
- $\frac{1}{\sqrt{1-\frac{2M}{r}}}$ is the gravitational blue-shift factor
- $\sqrt{\frac{1-v_l}{1+v_l}}$ is the Doppler shift factor
- a_p is... Unruh effect?

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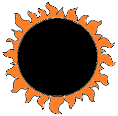
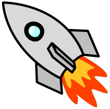
Sources of energy

- There are three sources of energy for the radiation detected:
  - The radiation emitted by the black hole



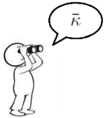
  - The energy provided by the observer's rockets

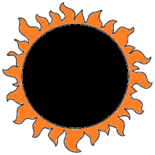


  - The gravitational potential energy of the observer

Asymptotic observers are privileged

- **Static observers in the asymptotic region** do not have rockets or exploit their gravitational energy
- These observers can only **detect radiation emission**
- But these observers **detect $\bar{\kappa}$!**





- $\bar{\kappa}$ discriminates the radiation emission (such as Hawking radiation)

Hawking versus Unruh

- $$\kappa(\tau) = \kappa_{\text{Hawk}}(\tau) + \kappa_{\text{Unruh}}(\tau)$$

- $$\kappa_{\text{Hawk}}(\tau) = \sqrt{\frac{1 - v_l}{1 + v_l}} \frac{1}{\sqrt{1 - \frac{2M}{r}}} \bar{\kappa}(\bar{u})$$

  This term is the radiation emitted to the asymptotic region, adequately shifted

- $$\kappa_{\text{Unruh}}(\tau) = -\sqrt{\frac{1 - v_l}{1 + v_l}} \frac{1}{\sqrt{1 - \frac{2M}{r}}} \frac{M}{r^2} + a_p$$

  This must be (by elimination) the Unruh effect

The Unruh effect as a relative acceleration

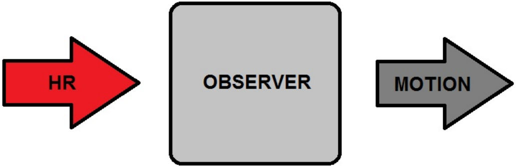
- Unruh effect contribution can be written as

$$\begin{aligned}
\kappa_{\mathrm{Unruh}}(\tau) &= -\left(\frac{\mathrm{d}\bar{u}}{\mathrm{d}\tau}\right)^{-1} \frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{\mathrm{d}\bar{u}}{\mathrm{d}\tau}\right) \\
&= -\left(\frac{\mathrm{d}\bar{u}}{\mathrm{d}\tau}\right)^{-1} \frac{\mathrm{d}}{\mathrm{d}\tau}\left(\sqrt{\frac{1-v_l}{1+v_l}}\frac{1}{\sqrt{1-\frac{2M}{r}}}\right)
\end{aligned}$$

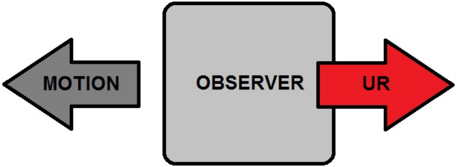
- This is the acceleration with respect to the asymptotic region, in the sense of the variation of the shift $\mathrm{d}\bar{u}/\mathrm{d}\tau$
- In Minkowski, this notion coincides with the proper acceleration

Different causes... and different consequences

- Each effect have **different consequences**
- Hawking radiation is external and produces **radiation action**



- Unruh effect produces itself Unruh radiation, which causes **radiation back-reaction**



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Static observers

- Unruh vacuum state with Hawking radiation:

$$\bar{\kappa} = \frac{1}{4M}$$

- An static observer at a radius r_0 perceives

$$\kappa = \kappa_{\mathrm{Hawk}} = \frac{1}{\sqrt{1 - \frac{2M}{r_0}}} \frac{1}{4M}$$

- This is just Hawking radiation with a gravitational blueshift
- The radiation pressure can lead to **buoyancy effects**

Free-falling observers

- An free-falling observer perceives

$$\kappa = \sqrt{\frac{1-v_l}{1+v_l}} \frac{1}{\sqrt{1-\frac{2M}{r}}} \left(\frac{1}{4M} - \frac{M}{r^2} \right)$$

- When approaching the horizon ($r \to 2M$)...

$$\frac{1}{\sqrt{1-\frac{2M}{r}}} \left(\frac{1}{4M} - \frac{M}{r^2} \right) \to 0$$

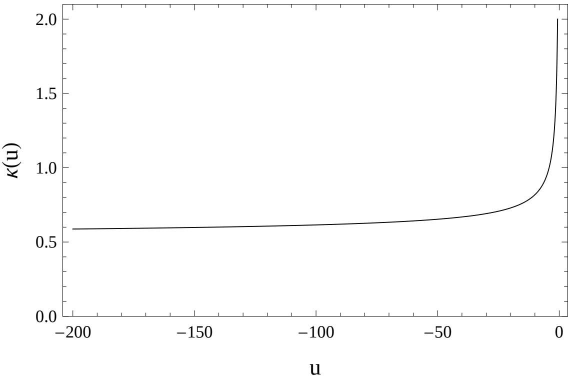
- But, along a given geodesic...

$$\kappa(\tau) \not\to 0$$

- This is due to a **diverging Doppler shift**
- For an observer left to fall from the asymptotic region

$$\kappa(\tau) \to \frac{1}{M}$$

Free-falling from infinity



Outline

It's not so easy to (slowly) cross the horizon

- For the ingoing radiation sector, we have

$$\kappa = \kappa_{\text{Unruh}} = \sqrt{\frac{1+v_l}{1-v_l}} \frac{1}{\sqrt{1-\frac{2M}{r}}} \frac{M}{r^2}$$

- When approaching the horizon ($r \to 2M$)...

$$\frac{1}{\sqrt{1-\frac{2M}{r}}} \frac{M}{r^2} \to \infty$$

- (Although, along a given geodesic...)

$$\kappa(\tau) \not\to \infty$$

- If you are free-falling (and slow), Unruh stops you
- If you are static (and close), Hawking stops you

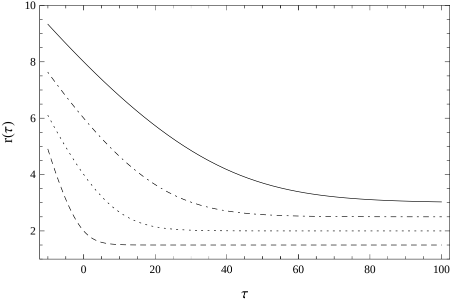
Outline

Quantum frictionless trajectories

- We want to have no Unruh effect in the outgoing sector
- The equation

$$\kappa_{\rm Unruh} = 0 \;\Rightarrow\; \sqrt{\frac{1-v_l}{1+v_l}}\frac{1}{\sqrt{1-\frac{2M}{r}}} = C$$

has many solutions: the **quantum frictionless** trajectories



Going into a buoyancy scenario

- These trajectories are not geodesic

$$a_p = \frac{CM}{r^2}$$

- If the object emits a total power P, the irradiance perceived is

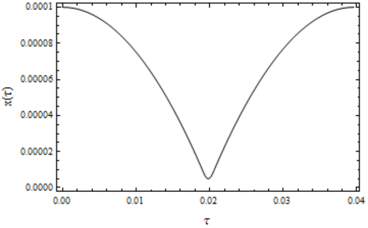
$$S = \frac{1-v_l}{1+v_l}\frac{1}{1-\frac{2M}{r}}\frac{P}{4\pi r^2} = \frac{C^2 P}{4\pi r^2}$$

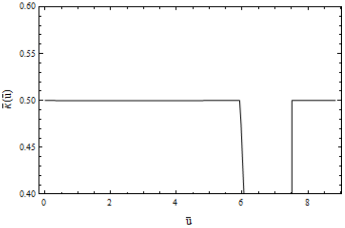
- The perceived radiation can be the source for the proper acceleration needed!
- There is ingoing Unruh effect, but it actually helps

Outline

The pulsating vacuum

- Let us consider a different vacuum: **the pulsating vacuum**





- There is no need to form a horizon
- There is no need to invoke trans-planckian frequencies

Outline

Relative effective temperatures

- Consider different families of observers, each of which naturally quantizes the field with particles corresponding to modes $\mathrm{e}^{-\mathrm{i}\omega u_i}$
- Consider the quantities:

$$D_{i,j} := \frac{\mathrm{d}u_j}{\mathrm{d}u_i}, \quad \kappa_{i,j} := -\left. \frac{\mathrm{d}^2 u_j}{\mathrm{d}u_i^2} \middle/ \frac{\mathrm{d}u_j}{\mathrm{d}u_i} \right. .$$

- We have the following relations:

$$\kappa_{i,j} = -D_{i,j}\kappa_{j,i}, \quad \kappa_{i,j} = D_{i,k}\kappa_{k,j} + \kappa_{i,k}$$

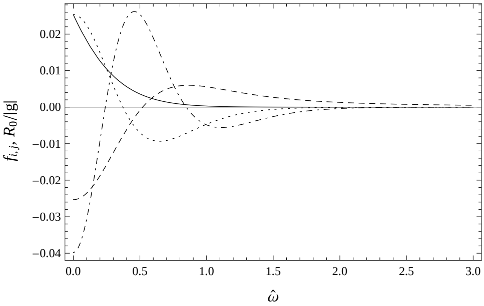
- The general expression for $\kappa(\tau)$ in Schwarzschild can be constructed out of these relations
- The key ingredient for the radiation perception is the relative variations of the shifts

Beyond the thermal spectrum

- Corrections to for non-constant (unidirectional) acceleration $g(\tau)$ (in $3+1$!)

$$\mathcal{R}_0 = |g(\tau)| \frac{\hat{\omega}}{2\pi} \frac{1}{e^{2\pi\hat{\omega}} - 1}, \quad \mathcal{R}_1 = g(\tau) \left[\frac{g'(\tau)}{g(\tau)^2} f_{1,1}(\hat{\omega}) \right],$$

$$\mathcal{R}_2 = |g(\tau)| \left[\frac{g''(\tau)}{g(\tau)^3} f_{2,1}(\hat{\omega}) + \frac{g'(\tau)^2}{g(\tau)^4} f_{2,2}(\hat{\omega}) \right].$$



- The construction should be reproduced in $1+1$ dimensions with $g(\tau) \to \kappa(\tau)$, and the factors changing

That's all...

Thanks for the attention!