Victoria University of Wellington

Te Whare Wānanga o te Ūpoko o te Ika a Maui

Observer-Dependent Tolman Temperatures

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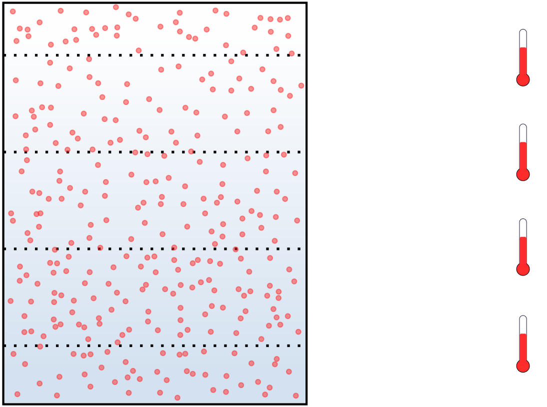
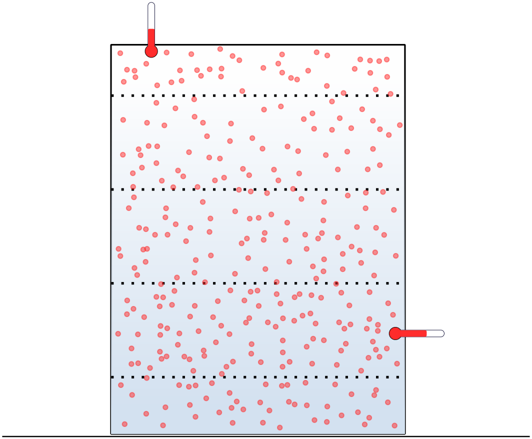
Seminar at Aotearoa Fundamental Physics Workshop 2018, Wellington
12th December 2018

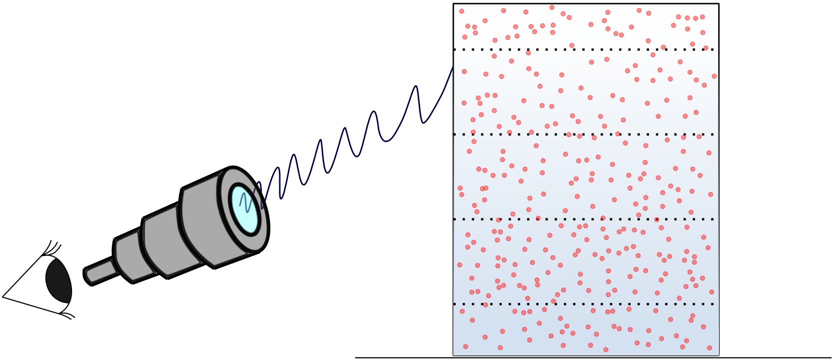
What is the temperature distribution of a gas ?



Local measurements



Non-local measurements



Outline

Equilibrium temperature gradients - static case

- Richard C. Tolman: *On the weight of heat and thermal equilibrium in General Relativity* (Phys. Rev. 35, 904 (1930)).

  Static spherically symmetric spacetime:

  $$ds^2 = g_{tt} dt^2 + g_{rr} dr^2 + r^2 \left(d\theta^2 + \sin^2 \theta d\phi^2 \right)$$

  + perfect fluid:

  $$T^{ab} = (\rho + p)\ V^a V^b + p\ g^{ab},$$

  with $V^a = (1,0,0,0)$.

- $\left(\nabla_a T^{ab} = 0 \right) + \left(G^{ab} = 8\pi T^{ab} \right)$ + assumptions depending on the fluid $\rightarrow$ temperature gradient.

  $$T = C\ \sqrt{-g^{tt}}$$

Temperature gradients in stationary spacetimes

Consider the relativistic Euler equation:

$$(\rho + p)A_a = -(\delta_a{}^b + V_a V^b)\nabla_b p.$$

For a photon gas [$\rho = aT^4$, $p = (a/3)T^4$] this simplifies to

$$A_a = -(\delta_a{}^b + V_a V^b)\nabla_b \ln T.$$

At thermal equilibrium $V^b \nabla_b T = 0$, so

$$A_a = -\nabla_a \ln T,$$

valid for any fluid in thermal equilibrium in a stationary spacetime.

Temperature gradients in stationary spacetimes

- Tolman's results can easily be recovered and extended for any *static spacetime* with its metric in the block-diagonal form:

$$ds^2 = g_{tt} dt^2 + g_{ij} dx^i dx^j,$$

where the 4-acceleration of the observers "at rest", $V^a \propto (1,0,0,0)$, is:

$$A_a = \nabla_a \ln \sqrt{-g_{tt}}.$$

Leading to

$$T(x) = T_0 \sqrt{-g^{tt}}$$

Temperature gradients in stationary spacetimes

- We can also calculate the 4-acceleration

$$A_a = \nabla_a \ln ||K||$$

of any observer following the integral curves an *arbitrary timelike* Killing vector

$$V^a = \hat{K}^a = \frac{K^a}{||K||}.$$

And consequently,

$$T(x) = \frac{T_0}{||K||}.$$

Temperature gradients in stationary spacetimes

- The other "natural" option is to take the fluid to follow a normal flow. Explicitly,

$$V^a = -\frac{\nabla^a t}{||\nabla t||};$$

where $||\nabla t|| = \sqrt{-g^{tt}}$.

Its 4-acceleration is:

$$A_a = -\nabla_a \ln ||\nabla t||.$$

Combined with $A_a = -\nabla_a \ln T$ this leads to

$$T(x) = T_0 \, ||\nabla t|| = T_0 \, \sqrt{-g^{tt}}.$$

$$T = T_0 \sqrt{-g^{tt}}$$

- At the flat earth approximation,

$$T(z) = T_0 \left\{1 - \frac{gz}{c^2}\right\},$$

giving us approximately

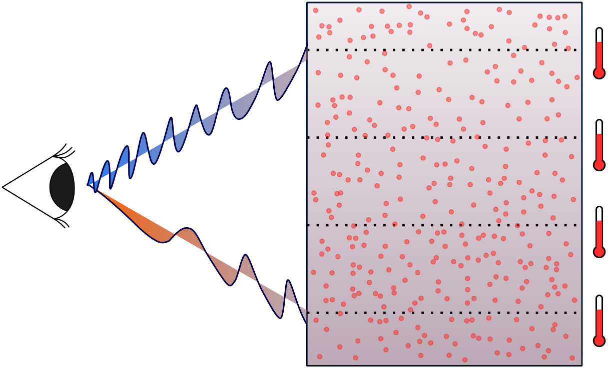
$$\frac{T(r)}{T_0} \approx \exp\left(-10^{-16} r \; m^{-1}\right)$$

What are the physical consequences?

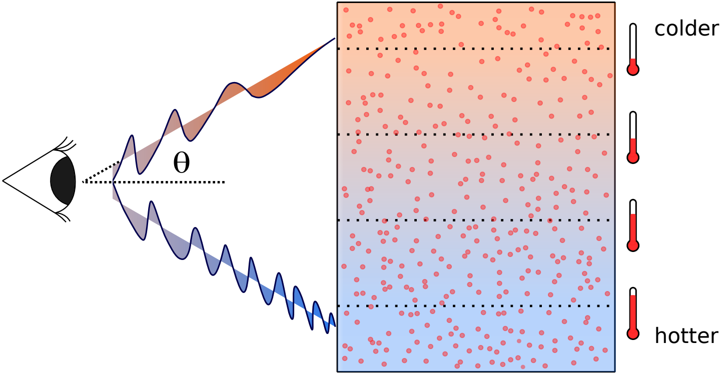
- Locally measured temperatures $T(x)$ have a small non-zero spatial gradient.

- Heat fluxes: Are driven by differences in the redshifted temperature T_0, *not* in the locally measured temperatures $T(x)$.

Constant local temperatures



Relativistic thermal equilibrium

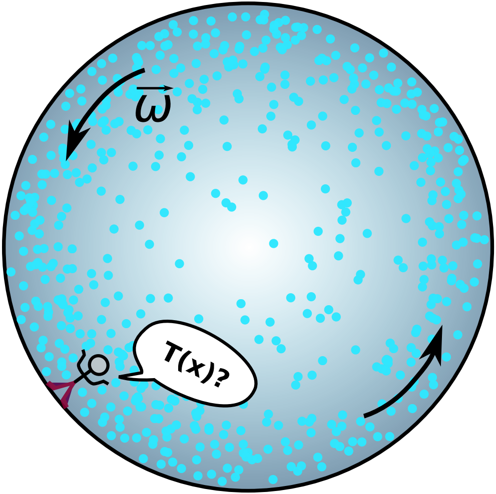


What are the physical consequences?

- Due to gravitational redshift, observers in a fluid's rest frame will see a constant temperature (T_0).
- Distinct observers will measure different values for T_0: *The temperature of a fluid in thermal equilibrium is observer dependent.*

Rotating cylinder example

What is the temperature distribution in a rotating frame?



The rotating frame temperature distribution

The metric seen by the rotating comoving observers is given by:

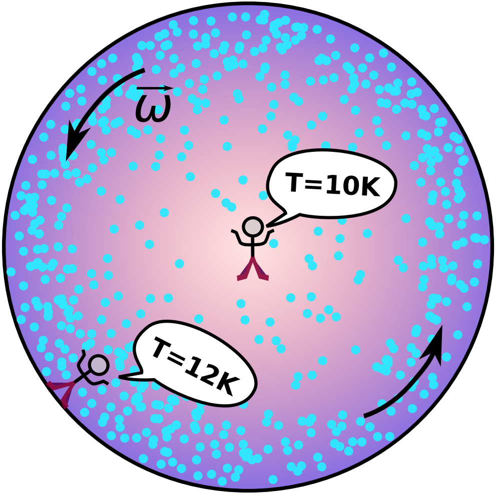
$$ds^2 = -dt^2 + dr^2 + r^2(d\phi - \omega dt)^2 + dz^2.$$

The co-rotating gas will follow trajectories of the Killing field $K^a = (1,0,0,0)$, with $||K|| = \sqrt{1-\omega^2 r^2}$ and $V^a = K^a/||K^a||$.

Applying this result to the stationary temperature gradients, we obtain:

$$T(x) = \frac{T_*}{||K||} = \frac{T_*}{\sqrt{1-\omega^2 r^2}}.$$

What happens for a non-inertial observer?

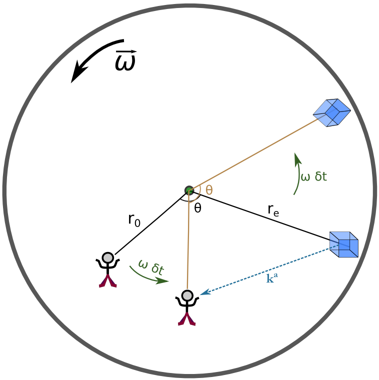


What is the blackbody spectrum seen by a comoving observer?

With V_e^a the 4-velocity of the emitter (thermal bath) and V_o^a the 4-velocity of the internal (comoving) observer, the redshift is given by

$$1 + z = \frac{(g_{ab} V_e^a k^b)_e}{(g_{ab} V_o^a k^b)_o} = \frac{\nu_e}{\nu_o}.$$

What is the blackbody spectrum seen by a comoving observer?



Let the emission event take place at: $X_e^a = (0,\, r_e,\, 0,\, 0)$

$$V_e = \gamma_e(1,\, 0,\, \omega r_e,\, 0,\, 0)$$

and the observation event at $X_o^a = (\delta t, r_o \cos\theta, r_o \sin\theta, 0)$

$$V_o = \gamma_o(1, -\omega r_o \sin\theta, \omega r_o \cos\theta, 0, 0).$$

Then

$$\delta X^a = (\delta t,\, r_o \cos\theta - r_e, r_o \sin\theta, 0).$$

What is the blackbody spectrum seen by a comoving observer?

The 4-vector of the light ray leaving the emitter and arriving at the observer is then

$$k^a = \frac{\delta X^a}{\delta t} = \left(1; \frac{r_o \cos\theta - r_e}{\delta t}, \frac{r_o \sin\theta}{\delta t}, 0\right).$$

Then we obtain:

$$(g_{ab} V^a k^b)_e = \gamma_e \left(-1 + \frac{\omega r_e r_o \sin\theta}{\delta t}\right),$$

and

$$(g_{ab} V^a k^b)_0 = \gamma_o \left(-1 + \frac{\omega r_e r_o \sin\theta}{\delta t}\right).$$

Consequently

$$1 + z = \frac{\gamma_e}{\gamma_o} = \sqrt{\frac{1 - \omega^2 r_o^2}{1 - \omega^2 r_e^2}}.$$

What is the black-body spectrum seen by a comoving observer?

As the black-body spectrum is emitted by the rotating gas in thermal equilibrium, we have

$$T(x_e) = \frac{T_*}{\sqrt{1 - \omega^2 r_e^2}}.$$

Being ν_* the maximum emission frequency at $r_e = 0$, the frequency at a random emission point is:

$$\nu_e = \frac{\nu_*}{\sqrt{1 - \omega^2 r_e^2}}.$$

What is the black-body spectrum seen by a comoving observer?

Given that $\nu_e/\nu_o = 1 + z$, we have

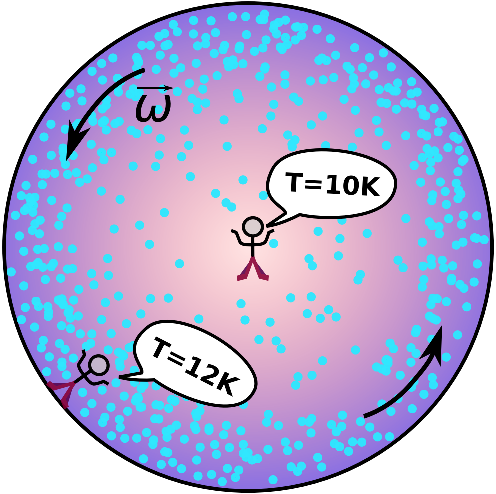
$$\nu_o = \frac{\nu_e}{1+z} = \frac{\nu_*}{\sqrt{1-\omega^2 r_e^2}} \sqrt{\frac{1-\omega^2 r_e^2}{1-\omega^2 r_o^2}} = \frac{\nu_*}{\sqrt{1-\omega^2 r_0^2}},$$

so the temperature seen by the observer is:

$$T(x_o) = \frac{T_*}{\sqrt{1-\omega^2 r_o^2}},$$

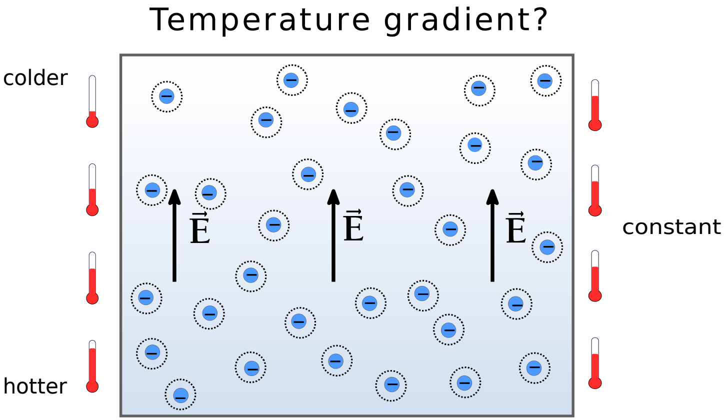
which is exactly the equilibrium temperature at the observer's location.

What happens for a non-inertial observer?

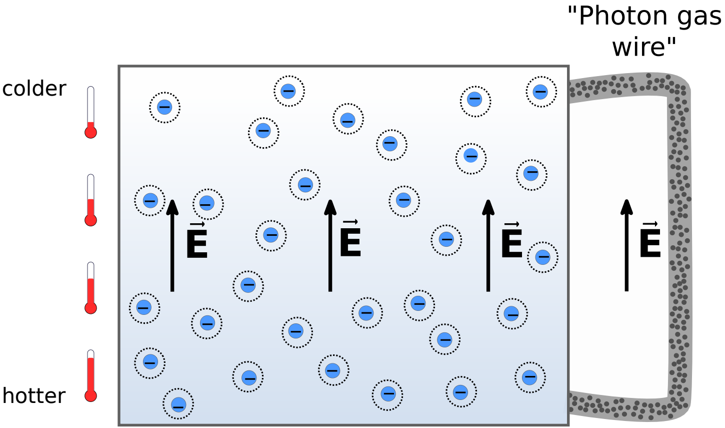


Electric Thermal Effects?

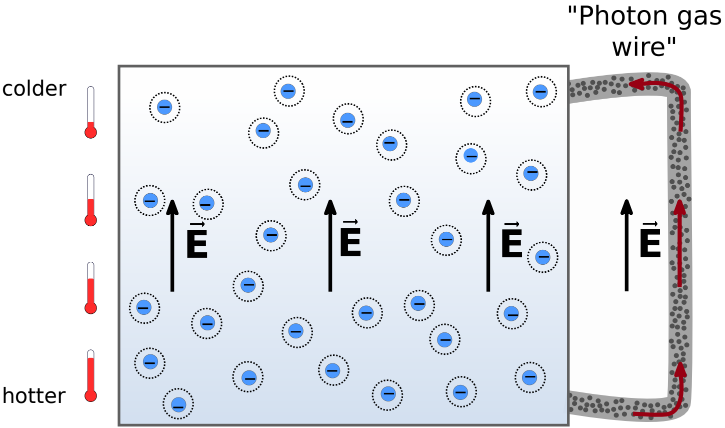
Equilibrium state in a electric field



Equilibrium state in a electric field



Equilibrium state in a electric field



So, can any other force do it?

Temperature gradients created by any force that is not universal (e.g. dependent of charge, mass, spin,...) would allow the creation of heat machines that violate the second law of thermodynamics.

For this reason they must not exist.

Gravity is the only force capable of creating temperature gradients in equilibrium states without violating any law of thermodynamics.

Current work

Can we still talk about equilibrium for non-Killing flows?

Observers following non-Killing trajectories experience a spacetime which is varying along their proper time.

On the other hand, it is important to define and talk about thermal equilibrium for a more general class of observers.

Can we push some limits and test until which point we can relax the Killing flow requirement without losing the idea of equilibrium?

What are the conditions for thermodynamical equilibrium?

Let $\mathfrak{S}(X_1, ..., X_i)$ be a system with 4-velocity u^a which can be fully characterized by a limited set of thermodynamic variables $\{X_j\}$. If $\mathfrak{S}$ is in thermodynamic equilibrium, then

- C-1: Constant thermodynamic variables

$$\mathfrak{L}_u \, X_j(x) = 0 \quad \text{or} \quad \langle \mathfrak{L}_u \, X_j(x) \rangle_\gamma = 0 \quad \forall j \in (1,i)$$

- C-2: Conserved currents

$$\nabla_a J^a = 0 \quad \text{with} \quad J_a = -T_{ab} \, u^b$$

- C-3: Invariant or almost invariant metric

$$\mathfrak{L}_u \, g^{ab} = 0 \quad \text{or} \quad \langle \mathfrak{L}_u \, g^{ab} \rangle_\gamma = 0.$$

The problem

I need to find a "rigorous enough" definition for $\langle \ \rangle_\gamma$.

If you have any ideas that can help me, let's talk :)

Thank you!!

Additional references

J. Santiago and M. Visser,
Tolman temperature gradients in a gravitational field;
arXiv:1803.04106 [gr-qc],

J. Santiago and M. Visser,
Gravity's universality: The physics underlying Tolman temperature gradients;
International Journal of Modern Physics D
DOI: 10.1142/S021827181846001X

J. Santiago and M. Visser,
Tolman-like temperature gradients in stationary spacetimes;
DOI: 10.1103/PhysRevD.98.064001