Second law non-violation theorem for Lorentz-noninvariant black holes

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Observer-dependent entropy

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R. Benkel, J. Bhattacharyya, JL, D. Mattingly, T. P. Sotiriou

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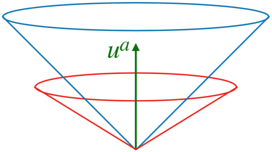

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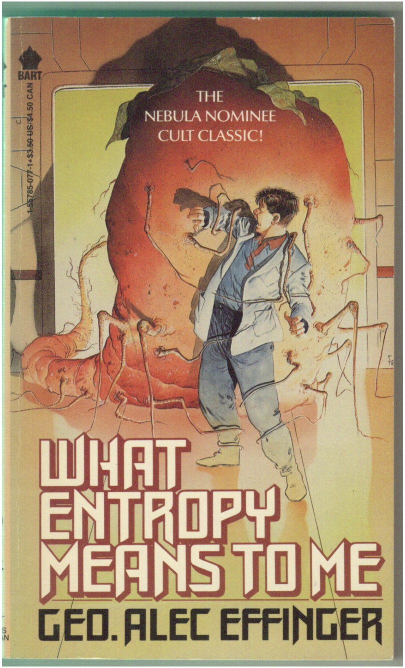
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THE
NEBULA NOMINEE
CULT CLASSIC!
WHAT
ENTROPY
MEANS TO ME
GEO. ALEC EFFINGER

Plan

1. **Einstein gravity Penrose process**
   - Splitting, collisions, tether...
2. **Covariant Lorentz violation**
   - Einstein-æther
3. **Lorentz-violating Penrose process**
   - **Spherical symmetry**
   - Splitting
4. **Results**
   - **Energy extraction admission theorem**
   - **Energy extraction no-go theorem**
5. **Upshots**

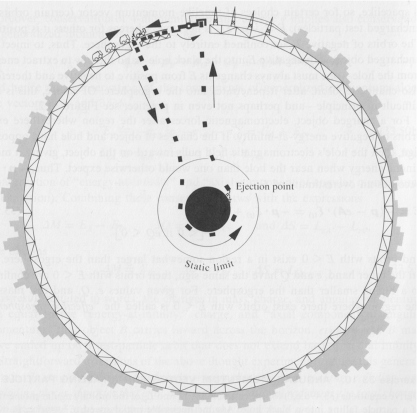
1. Einstein gravity Penrose process

Rotating black hole

Splitting version Penrose and Floyd 1971

1. Drop in shuttle
   + payload (waste)
2. Eject payload in ergoregion, *against* the rotation
3. Collect shuttle, extract energy from velocity

Extracted energy $> m_{\text{waste}} c^2$



Picture: Misner, Thorne and Wheeler 1973

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- Comes from rotational energy
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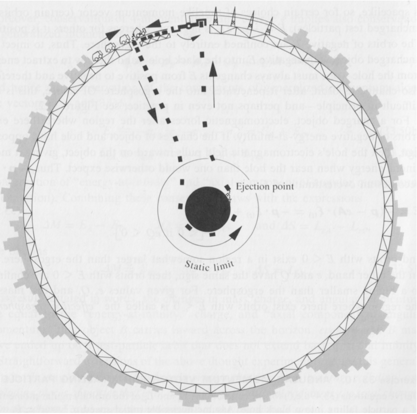

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- Energy budget drawn at infinity
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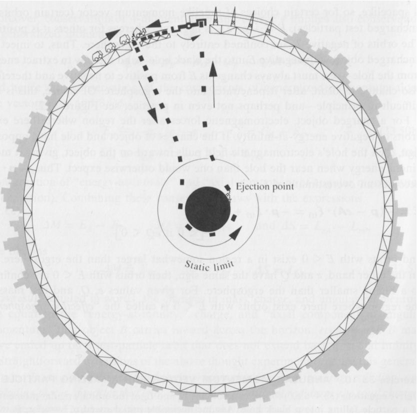
  $\rightarrow$ Laws of BH mechanics...
- Exists for $|J|/M^2 > 2/(\sqrt{2}+1)$
  Fayos Valles and Llanta Salleras 1991
  (and only for?)
- Collision version more efficient
  Wald 1974,...



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Einstein gravity Penrose process (cont'd)

Tether version Penrose 1969

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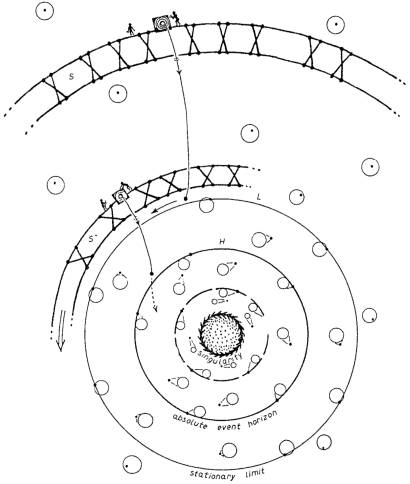

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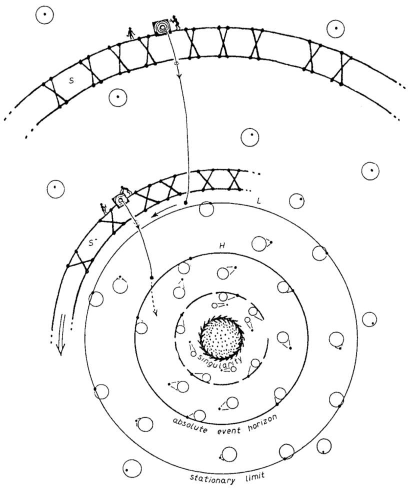
- Tether's net contribution to energy budget assumed negligible
  $\rightarrow$ Ongoing debate. . .
  Marolf and Sorkin 2002
  A. R. Brown 2013

Today: no tethers!



Picture: Penrose 1969

2. Covariant Lorentz violation: Einstein-æther

Fundamental Jacobson and Mattingly 2001,… or effective Hořava 2009,…

Dynamical fields:

- $g_{ab}^{(A)}$ $(-+++)$
- u^a with $u_a u^a = -1$ (æther)

  $\Rightarrow$ Distinguished timelike direction at each point

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$$g^{(B)}_{ab} = -u_a u_b + c^{-2}\big(g^{(A)}_{ab} + u_a u_b\big)$$

$$c > 1$$

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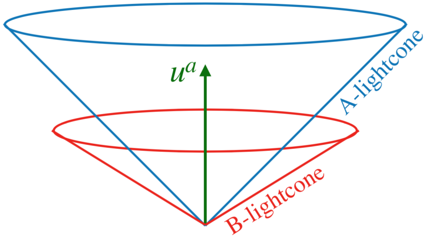

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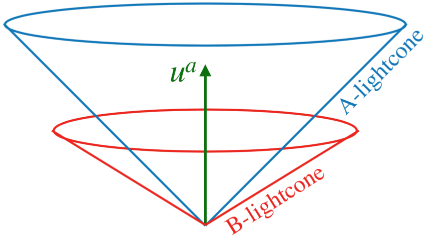
Excitations:

A-fields: hyperbolic in $g_{ab}^{(A)}$

B-fields: hyperbolic in $g_{ab}^{(B)}$

Local interactions



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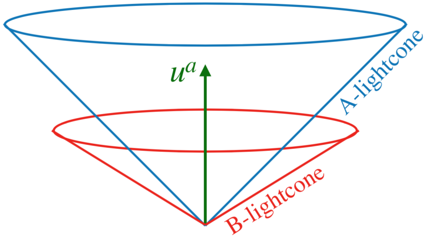
particles: geodesic
A-~~fields~~: ~~hyperbolic~~ in $g^{(A)}_{ab}$

particles: geodesic
B-~~fields~~: ~~hyperbolic~~ in $g^{(B)}_{ab}$



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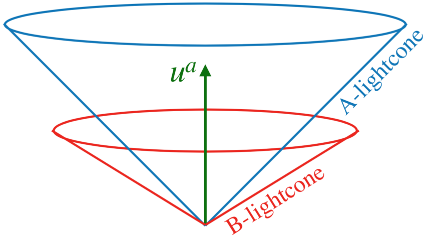


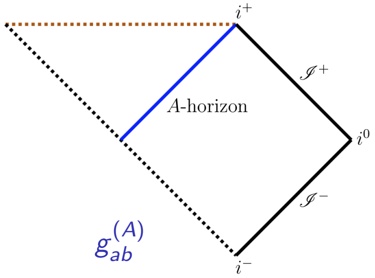
Local interactions

$\rightarrow$ **Collisions** conserving 4-momentum (1-form)

3. Lorentz-violating black hole

$g_{ab}^{(A)}$:

- static, spherically symmetric, asymptotically flat
- χ^a Killing, asymptotically Minkowski ∂_t at infinity
- future A-horizon: $\chi_a \chi^a$ changes sign



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- regular on A-horizon

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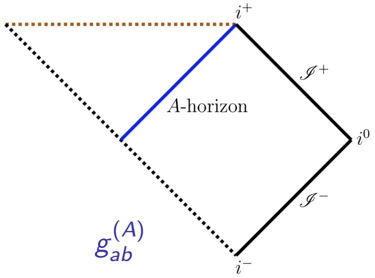

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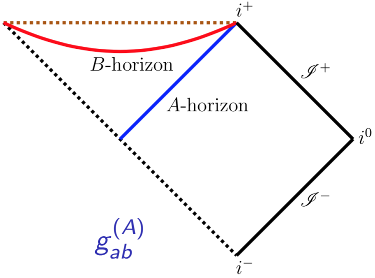

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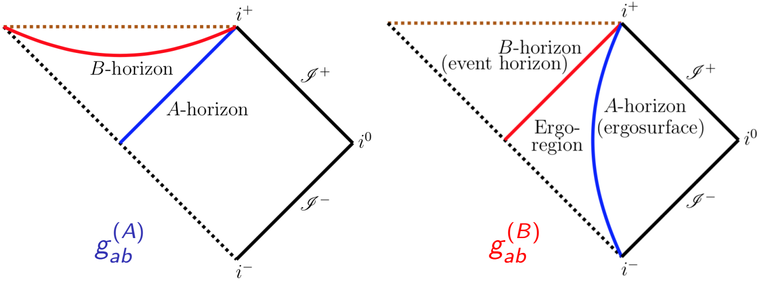
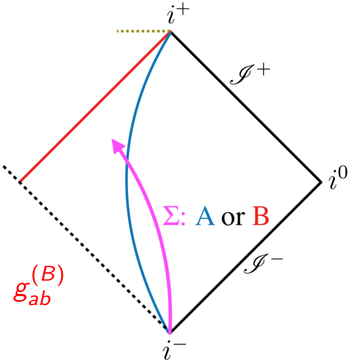


Penrose process

cf Eling et al 2007

Radial motion (by assumption)

- Σ (A or B) dropped from infinity



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- $\Sigma \to A + B$ split in ergoregion
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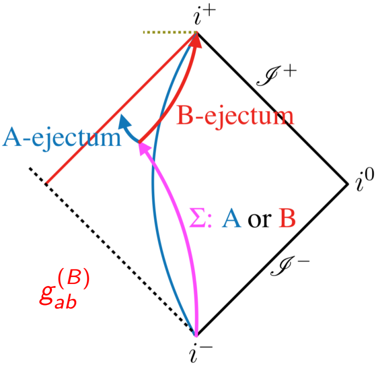

Killing energy at infinity?

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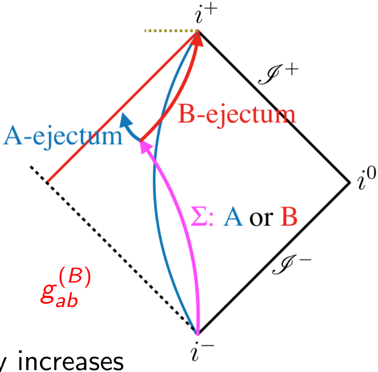

Killing energy at infinity?

Iff $-k_a^A \chi^a < 0$, Killing energy at infinity increases

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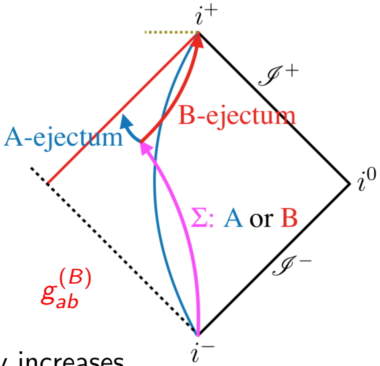

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$\Rightarrow$ End point of energy extraction?

$\Rightarrow$ Perpetual motion?

$\Rightarrow$ Violation of 2nd law of BH thermodynamics?!?

cf Eling et al 2007, Jacobson and Wall 2010, Dubovsky and Sibiryakov 2006

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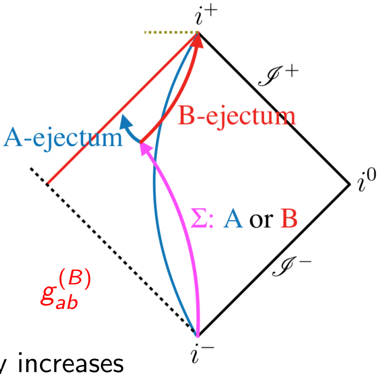

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For which $\left(g_{ab}^{(A)}, u^a\right)$ **does the process exist?**

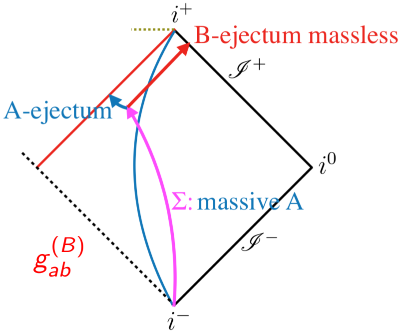
4. Results

1. Energy extraction admission theorem

For any $g_{ab}^{(A)}$, the process exists for **some** u^a

Construction:

- Σ: massive A
- B-ejectum massless



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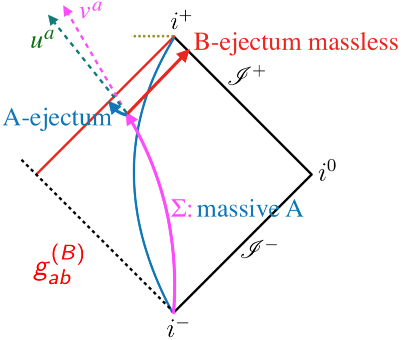

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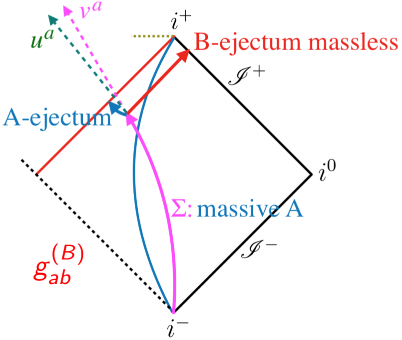

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Does this happen for 'reasonable' field equations?

4. Results (cont'd)

2. Energy extraction no-go theorem

If

$$-g_{ab}^{(B)}\chi^{a}\chi^{b}<1 \qquad (1)$$

in exterior $\cup$ ergosurface $\cup$ ergoregion, the process does not exist.

4. Results (cont'd)

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Comments

- Physics of (1): $-g^{(B)}_{00} < 1 \;\Rightarrow\;$ B-gravity attractive
- (1) implies $-g^{(A)}_{ab}\chi^a\chi^b < 1 \;\Rightarrow\;$ A-gravity attractive too
- (1) holds in **all** known Einstein-æther and Hořava solutions, analytic and numerical
- Might (1) necessarily follow from (reasonable) field equations?

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Proof: conceptually straightforward

5. Upshots

No-go theorem for Penrose splitting processes in spherically symmetric black holes without local Lorentz symmetry

- **Strong** despite limitations (e.g. radial motion)
  $\Rightarrow$ no perpetual motion
  $\Rightarrow$ no violation of 2nd law of thermodynamics

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Conjecture:

- If field equations allow $-g^{(B)}_{ab}\chi^a\chi^b < 1$ to be violated and energy extraction to occur, there must be new charges at infinity