Victoria University of Wellington

Te Whare Wānanga o te Ūpoko o te Ika a Maui

Observer dependent entropy (opening remarks)

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12–14 December 2018

Abstract:

- Entropy continues to be a very subtle concept.
  - Early versions of thermodynamic entropy were to a large extent "*objectively real*".
  - Modern versions of "coarse grained entropy" (either classical or quantum) have a *much more subtle ontology*...
- To what extent are modern notions of entropy observer dependent?
- To what extent are modern notions of temperature observer dependent?
- How does this impact on the laws of thermodynamics?
- How does this impact on Hawking radiation?

Topics:

Background

Background:

- Early versions of entropy, (Clausius $dS = đQ/T$, Carnot cycle, etc), were very *engineering-centric*...
- They were largely designed to answer the specific question: How much useful work can you get out of a specified mass of steam at specified temperature and pressure?

$$W = f(m, T, p)\ ?$$

- The engineering form of Clausius entropy is in some sense "*objectively real*", and not observer dependent...
- The ontological status of Shannon and von Neumann entropy is trickier...
- The ontological status of Bekenstein (black hole) entropy and Srednicki (closed box) entropy is much trickier...

Background:

- Clausius entropy:

$$dS = \frac{đQ}{T}$$

- (Carathéodory inexact differential.)
- Thence can define:

$$S = \int_\gamma \frac{đQ}{T}$$

- Need to define a suitable "zero" for the entropy.
- Need to define a suitable "path" γ.
- Leads to the well-developed theory of "classical thermodynamics".
- (Most of "*thermodynamics*" should really be called "*thermostatics*".)
- *Other types of entropy are much trickier*...

Background:

- Shannon entropy (classical):

$$S = -\sum_n p_n \ \ln p_n.$$

- *Probabalistic?*
- Subdivide each box into M sub-boxes of probability $p_{mn} = p_n/M$.
- Then

$$S' = -\sum_{m,n} p_{mn} \ \ln p_{mn} = -M \sum_n (p_n/M) \ \ln(p_n/M)$$

$$= -\sum_n p_n \ \ln p_n + \sum_n p_n \ \ln M = S + \ln M.$$

- That is

$$S' = S + \ln M$$

- *You can drive Shannon entropy arbitrarily large*, simply by splitting up the boxes...
- Observer dependent? (How close do you look?)

Background:

- von Neumann entropy (quantum):

$$S = -\mathrm{tr}\left\{\hat\rho\ \ln\hat\rho\right\}$$

- *Probabalistic?*
- Subdivide each dimension of the Hilbert space into M sub-dimensions with new density matrix $\hat\rho' = \hat\rho\otimes(I_M/M)$.
- Then

$$S' = -\mathrm{tr}'\left\{\hat\rho'\ \ln\hat\rho'\right\} = -\mathrm{tr}'\left\{[\hat\rho\otimes(I_M/M)]\ \ln[\hat\rho\otimes(I_M/M)]\right\}$$
$$= -M\ \mathrm{tr}\left\{[\hat\rho/M]\ \ln[\hat\rho/M]\right\} = -\mathrm{tr}\left\{\hat\rho\ \ln[\hat\rho/M]\right\}$$
$$= -\mathrm{tr}\left\{\hat\rho\ \ln\hat\rho\right\} + \mathrm{tr}\left\{\hat\rho\ \ln M\right\} = S + \ln M.$$

- That is

$$S' = S + \ln M$$

- *You can drive von Neumann entropy arbitrarily large*, simply by refining the Hilbert space...
- Observer dependent? (How close do you look?)

Background:

- Bekenstein entropy:

$$S = \frac{A}{4}$$

- Is the Bekenstein entropy:
  - Clausius like?

$$S = \int_\gamma \frac{đQ}{T}$$

    Take $Q = M$, and $T \propto 1/M$, (Einstein equations). Then $S \propto M^2 \propto A$. (With minor modifications also works for RN, Kerr, Kerr–Newman.)
  - von Neumann like?

$$S = -\mathrm{tr}\,\{\hat\rho\;\ln\hat\rho\}$$

    Entanglement? Srednicki argument? Horizon as "partial trace"?
- Still considerable confusion on this point...

Workshop goals

Workshop goals:

- To what extent are modern notions of entropy observer dependent?
- To what extent are modern notions of entropy objectively real?
- To what extent are modern notions of temperature observer dependent?
- To what extent are modern notions of temperature objectively real?
- How do these questions impact on the laws of thermodynamics?
- How do these questions impact on Hawking radiation?
    - Analogue Hawking radiation?
    - GR Hawking radiation from GR black holes?

Workshop goals:

Time	Wed 12/12	Thu 13/12	Fri 14/12
09:00-10:00	—	Rob	Jorma
10:00-11:00	—	Rob	Netta
11:00-11:30	*Tea/Coffee*	*Tea/Coffee*	*Tea/Coffee*
11:30-12:30	Matt	Jessica	Netta
12:30-14:00	*Lunch*	*Lunch*	*Lunch*
14:00-15:00	Matt	Luis	Valentina
15:00-16:00	Matt	Luis	Valentina
16:00-16:30	—	*Tea/Coffee*	*Tea/Coffee*
16:30-17:30	—	Jorma	Discussion & End
—	Cotton 119	Cotton 350	Cotton 350/119

Various entropies

Various entropies

Depending on context, entropy could mean:

- Clausius entropy:

$$\mathrm{d}S = \frac{\text{đ}Q}{T}; \qquad \Delta S = \int \frac{\text{đ}Q}{T}.$$

- Shannon entropy:
  - Continuum:

$$S = -\int \rho(x) \ln\left\{\frac{\rho(x)}{\rho_*}\right\} d^3x; \qquad \int \rho(x)\; d^3x = 1.$$

  - Discretium:

$$S = -\sum_i p_i \ln p_i; \qquad \sum_i p_i = 1.$$

- von Neumann entropy:

$$S = -\mathrm{tr}(\hat\rho \ln \hat\rho); \qquad \hat\rho \in (\text{Hermitian})^+; \qquad \mathrm{tr}(\hat\rho) = 1.$$

Context is important...

Clausius entropy

Clausius entropy:

$$\mathrm{d}S = \frac{đQ}{T}; \qquad \Delta S = \int \frac{đQ}{T}.$$

Heat flux divided by temperature...

- This definition most directly related to physics and engineering...
- Steampunk: How much useful work can you get out of a given mass of steam at specified temperature and pressure?
- Carnot cycles, heat engines, etc...
- Clausius entropy underlies "classical" thermodynamics; (*aka* Carathéodory thermodynamics).

Example: Blackbody furnace...

Photons emitted from a blackbody furnace are to an excellent approximation described by the Planck spectrum:

$$\frac{\mathrm{d}N}{\mathrm{d}\omega} = \frac{2}{(2\pi)^3} \, \frac{4\pi\omega^2}{\exp(\frac{\hbar\omega}{k_B T}) - 1}$$

Number of photons per unit frequency...

- $\hbar$ is Planck's constant.
- k_B is Boltzmann's constant.
- T is the (absolute) temperature.

This is early 1900's thermodynamics...
Solution to the "ultraviolet catastrophe"...
First introduction of Planck's constant...

- Each photon has energy

$$E = \hbar\omega$$

- Each photon carries a Clausius entropy

$$S = \frac{E}{T} = \frac{\hbar\omega}{T}$$

- On average

$$\langle S \rangle = \frac{\langle E \rangle}{T} = \frac{\hbar\langle\omega\rangle}{T}$$

- This is the *average Clausius entropy per photon*, emitted from a blackbody furnace at temperature T.
- Now calculate using the Planck spectrum...

- Calculate:

$$\langle \omega \rangle = \frac{\int_0^\infty \frac{\omega^3}{\exp(\hbar\omega/k_B T)-1}\, \mathrm{d}\omega}{\int_0^\infty \frac{\omega^2}{\exp(\hbar\omega/k_B T)-1}\, \mathrm{d}\omega} = \frac{\pi^4}{30\,\zeta(3)}\,\frac{k_B T}{\hbar}$$

- $\zeta(3)$ is Apery's constant, known to be irrational.
- Average energy

$$\langle E \rangle = \hbar \langle \omega \rangle = \frac{\pi^4}{30\,\zeta(3)}\, k_B T$$

- Average entropy

$$\langle S \rangle = \frac{\pi^4}{30\,\zeta(3)}\, k_B$$

- *Temperature drops out*.
- *You only need to know there is some well-defined temperature*, *you do not need to know its specific value*.

Clausius entropy

- In "natural units" (nats), $\hat{S} = S/k_B$ is dimensionless...

$$\langle \hat{S} \rangle = \frac{\pi^4}{30\,\zeta(3)}$$

- The equivalent number of bits, using Boltzmann's magic formula

$$S = \ln \Omega = \ln(2^{\{bits\}}) = \{bits\} \ln 2; \qquad \hat{S}_2 = \frac{\hat{S}}{\ln 2}$$

- Then

$$\langle \hat{S}_2 \rangle = \frac{\pi^4}{30\,\zeta(3) \ln 2} \approx 3.896976153 \;\; \text{bits/photon.}$$

- Every photon in this room will on average carry 3.89 bits of entropy.
- (Or more if it's not blackbody...)
- Potential context dependence of Clausius entropy?

- *Completely non-controversial*...
- Entropy is hiding in the correlations...
- See for instance:
  "On burning a lump of coal",
  Ana Alonso-Serrano and Matt Visser,
  Phys. Lett. B **757** (2016) 383
  doi:10.1016/j.physletb.2016.04.023 [arXiv:1511.01162 [gr-qc]].
- As long as the burning process is "adiabatic"
  (temperature slowly changing compared to the average frequency)

$$\frac{\dot{T}}{T} \ll \langle \omega \rangle,$$

then (in bits)

$$\Delta S \sim \frac{\pi^4}{30\,\zeta(3)\ln 2}\, N_{\mathrm{photons}}.$$

Clausius entropy

- The existence of Clausius entropy is ultimately due to the fact that one cannot track all the individual molecules in the steam...
- (Or the individual photons in the blackbody radiation...)
- You can only observe aggregates (total mass), and averages (pressure, temperature).
- Aggregates $\Longrightarrow$ Extensive variables...
- Averages $\Longrightarrow$ Intensive variables...
- This immediately leads to the notion of statistical mechanix...
- So let's jump straight to (classical) statistics; (classical) probability distributions, and Shannon entropy.
- (Quantum statistics; density matrices; and von Neumann entropy will be dealt with later...)

Shannon entropy

For any normalized probability distribution the Shannon entropy is:

- Continuum:

$$S(\rho,\rho_*) = -\int \rho(x)\ln\left\{\frac{\rho(x)}{\rho_*}\right\}\, d^3x; \qquad \int \rho(x)\, d^3x = 1.$$

Here ρ_* is a *fixed-but-arbitrary* normalization parameter;
just don't change it in the middle of the calculation...
It is annoying, but it is essential to keep track of it..

- Discretium:

$$S(p) = -\sum_i p_i \ln p_i; \qquad \sum_i p_i = 1.$$

Directly relevant to classical communication channels.
Computer scientists and communications engineers like to use $\log_2 p_i$, and talk about entropy (or information) in bits; physicists like to use $\ln p_i$ and talk about entropy in nats.

Continuum Shannon entropy

Continuum Shannon entropy

Example: Gaussian distribution

- Probability density (1-d)

$$\rho_\sigma(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-x^2}{2\sigma^2}\right)$$

- *Shannon entropy of a Gaussian distribution:*

$$S(\rho_\sigma, \rho_*) = -\int \rho(x) \ln\left\{\frac{\rho(x)}{\rho_*}\right\} dx = \frac{1}{2} + \ln\sqrt{2\pi} + \ln(\sigma\rho_*)$$

- The *physically interesting quantity is the Shannon entropy difference*:

$$S(\rho_{\sigma_1}, \rho_*) - S(\rho_{\sigma_2}, \rho_*) = \ln(\sigma_1/\sigma_2)$$

- Larger standard deviation ⇒ greater uncertainty ⇒ higher entropy...
- Normalization ρ_* drops out of the entropy difference...

Continuum Shannon entropy

Example: Blurred vision

- Suppose you have an initial probability distribution $\rho(x)$, and then suffer from some form of "blurred vision" so that

$$\rho(x) \to \int_{-\infty}^{+\infty} K(x,y)\rho(y)\mathrm{d}y; \qquad \int_{-\infty}^{+\infty} K(x,y)\mathrm{d}x = 1.$$

- How does this affect the entropy?
- Specific model: *Gaussian blurring*:

$$K(x,y) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-(x-y)^2}{2\sigma^2}\right)$$

- Note:

$$\partial_\sigma K(x,y) = \sigma\, \partial_x^2 K(x,y)$$

- Formally related to diffusion...

Example: Blurred vision

- Start calculating:

$$S(\rho_K) = -\int \rho_K(x) \; \ln\left(\frac{\rho_K(x)}{\rho_*}\right) \; \mathrm{d}x$$

$$\partial_\sigma S(\rho_K) = -\int (\partial_\sigma \rho_K(x)) \; \ln\left(\frac{\rho_K(x)}{\rho_*}\right) \; \mathrm{d}x$$

$$\partial_\sigma S(\rho_K) = -\int \sigma \; (\partial_x^2 \rho_K(x)) \ln(\rho_{K_\sigma}/\rho_*) \; \mathrm{d}x$$

$$\partial_\sigma S(\rho_K) = \sigma \int \frac{(\partial_x \rho_K(x))^2}{\rho_K(x)} \; \mathrm{d}x \geq 0$$

- Closely related to but not quite the Fisher information (as a function of the blurring parameter)...

Example: Blurred vision

- Blurred vision always increases Shannon entropy...
- Blink, refocus, then Shannon entropy decreases...
- Shannon entropy is contextual and can be observer dependent...
- Entropy rising?
  - Not quite always...
  - Depends:
    - Is the diffusion physical?
    - Or is it a gedanken-process?
- Think of blurred vision as a (reversible) coarse-graining...
- Think of physical diffusion as an (irreversible) coarse-graining...
- Coarse-graining much less well-understood than people like to think...
- Shortage of fully explicit calculable tuneable models...

For some specific models of coarse-graining see:

- "Coarse graining Shannon and von Neumann entropies",
  Ana Alonso-Serrano and Matt Visser,
  Entropy **19** (2017) # 5, 207
  doi:10.3390/e19050207 [arXiv:1704.00237 [quant-ph]].

Continuum Shannon entropy

Example: Fisher information

- Interpolating densities:

$$\rho_s(x) = (1-s)\rho_1(x) + s\rho_2(x)$$

$$S(\rho_s) = -\int \rho_s(x) \ln\left(\frac{\rho_s(x)}{\rho_*}\right) \mathrm{d}x$$

$$\partial_s S(\rho_s) = -\int (\partial_s \rho_s(x)) \ln\left(\frac{\rho_s(x)}{\rho_*}\right) \mathrm{d}x$$

$$\partial_s^2 S(\rho_K) = -\int \frac{(\partial_s \rho_s(x))^2}{\rho_s(x)} \mathrm{d}x \leq 0$$

- This is the Fisher information (as a function of the interpolating parameter)...
- Implies concavity of the Shannon entropy...

Discretium Shannon entropy

Discretium Shannon entropy

- If state-space is finite, $i \in [1..N]$, then

$$S(p) = -\sum_{i=1}^{N} p_i \ln p_i \leq \ln N.$$

- More generally if $N_* = \#\{i : p_i > 0\}$ then

$$S(p) = -\sum_{i} p_i \ln p_i \leq \ln N_*.$$

- Even more generally:

$$S(p) = -\sum_{i} p_i \ln p_i \leq -\ln \inf(p_i > 0).$$

- One can get infinite Shannon entropy by suitably dispersing a finite amount of probability into an infinite number of states...
- Many powerful theorems... (Mainly based on using real analysis.)

For some specific examples see:

- "Infinite Shannon entropy",
  Valentina Baccetti and Matt Visser,
  Journal of Statistical Mechanics: Theory and Experiment
  2013 (2013) P04010
  doi: 10.1088/1742-5468/2013/04/P04010
  arXiv:1212.5630 [cond-mat.stat-mech]

Discretium Shannon entropy

Example:

- Start from the continuum.
- Divide the universe up into a denumerable set of boxes B_i.
- Equal volumes for simplicity.
- Define

$$p_i = \int_{B_i} \rho(x) d^3x,$$

- You are agreeing not to look at detailed information of the probability distribution inside each individual box.
- Compare the continuum and discretium entropies

$$S(\rho, \rho_*) = -\int \rho(x) \ln\left\{\frac{\rho(x)}{\rho_*}\right\} d^3x; \quad \text{and} \quad S_B = -\sum p_i \ln p_i.$$

- Define a box-wise constant function

$$\rho_B(x) : \text{if } x \in \mathrm{int}(B_i) \text{ then } \rho_B(x) \;= \frac{p_i}{V} = \frac{\int_{B_i} \rho(x)\; d^3x}{\int_{B_i}\; d^3x}.$$

- Invoke relative entropy inequality

$$\int \rho(x) \ln\left\{\frac{\rho(x)}{\rho_B(x)}\right\} d^3x \geq 0.$$

- Since ρ_B is box-wise constant

$$-\int \rho(x) \ln\left\{\frac{\rho(x)}{\rho_*}\right\} d^3x \leq -\int \rho_B(x) \ln\left\{\frac{\rho_B(x)}{\rho_*}\right\} d^3x.$$

- Entropy rises:

$$S(\rho, \rho_*) \leq S(\rho_B, \rho_*).$$

- But wait:

$$S(\rho_B, \rho_*) = -\int \rho_B(x) \ln\left\{\frac{\rho_B(x)}{\rho_*}\right\} d^3x = -\sum p_i \ln\left(\frac{p_i}{\rho_* V}\right)$$

$$= -\sum p_i \ln p_i + \ln(\rho_* V) = S_B + \ln(\rho_* V)$$

- That is, entropy rises:

$$S_B = S(\rho_B, \rho_*) - \ln(\rho_* V) \geq S(\rho, \rho_*) - \ln(\rho_* V)$$

- *If you agree to not look inside the individual boxes, then the entropy increases*...
- *If you change your mind, and look inside the boxes, then the entropy decreases*...

- Many extensions of these ideas...
- Apply Gaussian blurring to $\rho(x)$ before box-averaging...
  - Then the boxed entropy depends continuously and monotonically on the blurring parameter...
- Aggregate/average the boxed probabilities. Take two boxes and set:

$$p_{a,new} = p_{b,new} = \bar{p} = \frac{p_a + p_b}{2}.$$

  - Then the boxed entropy is non decreasing under aggregation/ averaging...
- Overall, coarse-graining always increases entropy...
- If this is a gedanken-process, then the coarse graining is reversible, and entropy can decrease...
- Entropy an be context-dependent and observer-dependent.

von Neumann entropy

von Neumann entropy

The von Neumann (quantum) entropy:

$$S = -\mathrm{tr}(\rho \ln \rho); \qquad \rho \in (\text{Hermitian})^{+}; \qquad \mathrm{tr}(\rho) = 1.$$

- Almost always physicists will immediately simplify things by going to finite-dimensional Hilbert space.

$$S(\rho) \leq \ln N.$$

- Large but finite: $N \gtrsim \exp(2 \times 10^{77}) \approx 10^{10^{77}}$ is not uncommon.
- Be thankful for small mercies, not quite a googolplex!

- The density matrix ρ is Hermitian positive semidefinite.
- The density matrix ρ generalizes the classical notion of probability:

$$\rho = U \,\mathrm{diag}\{p_i\}\, U^\dagger; \qquad p_i \geq 0; \qquad \sum_i p_i = 1.$$

- But wait:

$$S = -\mathrm{tr}(\rho \ln \rho) \quad \& \quad \rho = U \,\mathrm{diag}\{p_i\}\, U^\dagger \quad \Rightarrow \quad S = -\sum_i p_i \ln p_i.$$

- This is the formula for Shannon entropy! So what is new?
- If you compare/contrast two distinct density matrices, then they need not commute...

von Neumann entropy

- Very roughly speaking, the non-commuting nature of position and momentum, the Heisenberg uncertainty principle, can eventually render quantum probabilities non-commuting...
- Dealing with quantum (rather than classical) probabilities is much more technically involved; you need to work with operator algebras; and morphisms and functions on operator algebras...
- Often results for classical Shannon entropy carry over (with a lot more work) to the quantum von Neumann entropy...
- Sometimes the quantum von Neumann entropy exhibits radically different behaviour...
    - Sub-additivity...
    - Strong sub-additivity...

Example:

- Hawking's super-scattering operator is a linear mapping from density matrices to density matrices:

$$\rho \quad \to \quad \$\, \rho \quad \neq \quad S\, \rho\, S^\dagger$$

- Warning: Terminology inconsistent:
  Hawking super-scattering operator also known as:
  — "trace-preserving (completely) positive operator",
  — "quantum map",
  — "quantum process",
  — "quantum channel".
- Usage (and precise definition) is (unfortunately) not entirely standardized.

- Pick a super-scattering operator such that

$$S(\$\rho) \geq S(\rho)$$

- Many examples of this phenomenon are known...
  (Decoherence, maximal mixing)
- Now consider:

$$\rho \to \rho_s = e^{-s[I-\$]}\rho = e^{-s} e^{s\$} \rho$$

- This satisfies (shown below)

$$S(\rho_s) \geq S(\rho)$$

- Entropy rises...
- Coarse-graining...
- The process $\rho_s = e^{-s} e^{s\$} \rho$ represents "*diffusion on Hilbert space*"...
- Not a diffusion on real physical 3-space...

- Definition:

$$S(\rho_s) = -\text{tr}(\rho_s \ln \rho_s)$$

- Calculate:

$$\partial_s S(\rho_s) = -\text{tr}((\partial_s \rho_s) \ln \rho_s) = -\text{tr}((-\rho_s + \$\rho_s) \ln \rho_s)$$

$$\partial_s S(\rho_s) = -S(\rho_S) - \text{tr}((\$\rho_s) \ln \rho_s)$$

- Apply quantum relative entropy inequality

$$\partial_s S(\rho_s) \geq -S(\rho_S) - \text{tr}((\$\rho_s) \ln(\$\rho_s))$$

$$\partial_s S(\rho_s) \geq S(\$\rho) - S(\rho_S) \geq 0$$

- Entropy rises...
- Coarse-graining with tuneable parameter...

- Can think of $\Delta = \$ - I_N$ as a "Laplacian" on Hilbert space...
- For large s you are driven to the "ground state"

$$\Delta \rho_\infty = 0; \qquad \$ \rho_\infty = \rho_\infty$$

- But $S(\$\rho) \geq S(\rho)$ by hypothesis...
- Maximal mixing:

$$S(I_N/N) = \ln N.$$

- So $\rho_\infty = I_N/N$ is the maximal mixing (maximum entropy) state...
- That is

$$\lim_{s\to\infty} e^{s\Delta} \rho = \lim_{s\to\infty} e^{-s} e^{s\$} \rho = \frac{I_N}{N}$$

For some specific models of coarse-graining see:

- "Coarse graining Shannon and von Neumann entropies",
  Ana Alonso-Serrano and Matt Visser,
  Entropy **19** (2017) # 5, 207
  doi:10.3390/e19050207 [arXiv:1704.00237 [quant-ph]].

Summary so far

Summary so far:

- Entropy rises, except when it doesn't...
- Entropy can be both context dependent and observer dependent...
- Coarse-graining can be physical and irreversible;
  or a reversible gedanken-process...
- Coarse-graining is *nowhere near as well-understood* as people would like to think...
- Central to understanding the Hawking evaporation of black holes...
- Lots of tricky and subtle mathematics and physics involved...

Workshop goals reboot

Workshop goals reboot:

- To what extent are modern notions of entropy observer dependent?
- To what extent are modern notions of entropy objectively real?

- To what extent are modern notions of temperature observer dependent?
- To what extent are modern notions of temperature objectively real?
- How do these questions impact on the laws of thermodynamics?

- How do these questions impact on Hawking radiation?
  - Analogue Hawking radiation?
  - GR Hawking radiation from GR black holes?

Hawking radiation

Hawking radiation:

- The relationship between coarse-graining and Bekenstein entropy is a subtle one.
- For instance, the gravitational collapse that forms a black hole can be interpreted as an extreme form of coarse graining, as the region behind the horizon becomes, (either temporarily or permanently), inaccessible.
    - But is this coarse-grained entropy objectively "real"?
    - Or is it a "virtual" gedanken-entropy, reversible once one looks behind the horizon?
    - Is it synonymous with the Bekenstein entropy?
    - And how does it relate to the "information puzzle"?

Planckian versus thermal

Planckian versus thermal:

- Planckian is not exactly the same as thermal...
- Planckian is simply a statement about shape of the spectrum...
- Thermal implies something more about the correlations, or lack of correlations...
- Blackbody radiation, (in the traditional statistical mechanics sense), implies there must be correlations, simply because traditional statistical mechanics is unitary.
- The assumed lack of correlations in Hawking radiation is an artefact of assuming event horizons...
- With long-lived apparent horizons there can be correlations.

See for example:

- "Thermality of the Hawking flux",
  JHEP **1507** (2015) 009
  doi:10.1007/JHEP07(2015)009 [arXiv:1409.7754 [gr-qc]].
- The difference between Planckian and thermal is not controversial...
- However, endless confusion still abounds...
- *Treat the phrase "event horizon" with extreme caution*...

Hawking radiation is pure kinematics

Hawking radiation is pure kinematics:

- Hawking radiation is pure kinematics...
  - QFT plus apparent horizon (surface gravity) is all you need...

$$k_B\, T_H = \frac{\hbar\, \kappa_H}{2\pi\, c_H}.$$

- Bekenstein entropy directly related to Einstein dynamics...
  - Integrate the Clausius relation

$$dS = \frac{dE}{T_H}.$$

  - Apply Jacobson 1995 argument...
  - Yes, entropic forces are certainly real (and reversible)...

$$F = \frac{dE}{dx} = T_H\, \frac{dS}{dx}.$$

Hawking radiation is pure kinematics:

See for instance:

- "Analogue gravity",
  Carlox Barceló, Stefano Liberati, and Matt Visser,
  Living Rev. Rel. **8** (2005) 12 [Living Rev. Rel. **14** (2011) 3]
  doi:10.12942/lrr-2005-12 [gr-qc/0505065].
- Analogue spacetimes,
  (acoustics, surface waves, optical solitons, BECs, etc, etc),
  let you have Hawking radiation without Bekenstein entropy.
- *Some experiments already done*...
- *More experiments on the way*...

The big coarse graining

The big coarse graining:

- There is a surprising amount of confusion as to what the entropy of a "young" black hole is just after collapse:
  - Bekenstein would say (entropy)=(area)/4.
  - Strominger–Vafa would say (entropy)=(area)/4.
  - Srednicki would say (entropy) $\propto$ (area).
  - Bombelli–Sorkin would say (entropy)=(area)/4,
  - Wald would say (entropy)=(area)/4,
  - But the pro-firewall paradoxers say (entropy)=0.
- *Confusion traces back to the question of just how you coarse grain, (or refuse to coarse grain), during the collapse process*...

The big coarse graining:

- Horizon formation is a big coarse graining event:
  - Bekenstein entropy counts the number of ways the black hole *could have formed*...
  - Known for 40 years or more...
  - Ignoring Bekenstein entropy during the slow evaporation phase quickly leads to gibberish...
  - *(This point obvious but nevertheless controversial...)*

The big coarse graining:

- *You really should perform a tri-partite entropy budget, not a bi-partite entropy budget.*
- That is, analyze:
  (Black hole)+(Hawking radiation)+(Rest of universe).
- If you just use:
  (Black hole)+(Hawking radiation)
  then you simply cannot handle the Bekenstein entropy.
- *(The pro-firewall enthusiasts really did not want to hear this...)*

The big coarse graining:

See for instance:

- "Entropy/information flux in Hawking radiation",
  Ana Alonso-Serrano and Matt Visser,
  Phys. Lett. B **776** (2018) 10
  doi:10.1016/j.physletb.2017.11.020
  [arXiv:1512.01890 [gr-qc]].
  (Physics obvious but nevertheless controversial...)
  (At least among the pro-firewall enthusiasts...)
- "Entropy budget for Hawking evaporation",
  Ana Alonso-Serrano and Matt Visser,
  Universe **3** (2017) #3 58
  doi:10.3390/universe3030058
  [arXiv:1707.07457 [gr-qc]].
  (Physics obvious but nevertheless controversial...)
  (At least among the pro-firewall enthusiasts...)

The big coarse graining:

See for instance:

- "Multipartite analysis of average-subsystem entropies",
  Ana Alonso-Serrano and Matt Visser,
  Phys. Rev. A **96** (2017) #5, 052302
  doi:10.1103/PhysRevA.96.052302
  [arXiv:1707.09755 [quant-ph]].
  (Physics utterly non-controversial...)
  (Note PRA not PRD...)
- "Gravitational collapse: The big coarse-graining",
  Ana Alonso-Serrano and Matt Visser,
  (in preparation; hopefully to appear sometime this decade...)

Information puzzle

Information puzzle:

- The information puzzle is an artefact of extrapolating general relativity all the way up to the Planck scale...
- The information puzzle depends on *near-singularity physics* ...
- Hawking radiation only cares about apparent/trapping horizons...
- Hawking radiation does not care about event horizons...
- Event horizons are an artefact of extrapolating general relativity all the way up to the Planck scale...
- Existence of event horizons depends on *near-singularity physics*...
- Even Stephen Hawking has abjured event horizons — twice...
- Event horizons are simply not (empirical) physics...

Hawking radiation without event horizons:

- See Ashtekar–Bojowald, Hayward, Bardeen, Frolov, or (modified)-Bergmann–Roman pictures for how to have Hawking radiation without event horizons...

See for instance:

- "*On the viability of regular black holes*"
  R. Carballo-Rubio, F. Di Filippo, S. Liberati, C. Pacilio and M. Visser,
  JHEP **1807** (2018) 023 [JHEP **2018** (2020) 023]
  doi:10.1007/JHEP07(2018)023 [arXiv:1805.02675 [gr-qc]].
- "*Phenomenological aspects of black holes beyond general relativity*",
  R. Carballo-Rubio, F. Di Filippo, S. Liberati and M. Visser,
  Physical Review D **98** (2018) 124009.
  doi: 10.1103/PhysRevD.98.124009 [arXiv:1809.08238 [gr-qc]].

Information puzzle:

- Event horizons are mathematically convenient for proving theorems...
- Event horizons cannot, (neither their presence nor their absence), ever be detected via finite-size finite-duration experiments...
- Teleology can be good mathematics, but it is generally bad physics...
- Apparent/trapping horizons, (either their presence or their absence), can (at least in spherical symmetry) be detected via finite-size finite-duration experiments...
- (quasi-local physics *versus* ultra-local physics)...
- No event horizon, no (intrinsic) information puzzle...
- No event horizon, still desirable to calculate entropy fluxes...
- Event horizons are simply not (empirical) physics...

See for example:

- "Physical observability of horizons",
  Matt. Visser,
  Phys. Rev. D **90** (2014) no.12, 127502
  doi:10.1103/PhysRevD.90.127502 [arXiv:1407.7295 [gr-qc]].
- *The non-empirical nature of event horizons is not controversial*...
- Apparent/trapping horizons much better in this regard...
- Physics not controversial...
- *Endless confusion still abounds*...

Information puzzle:

- Unitarity preserving Planck spectra encode about 4 bits/photon in the correlations...
- More precisely:

$$\langle \hat{S} \rangle = \frac{\langle E \rangle}{k_B T} = \frac{\hbar \langle \omega \rangle}{k_B T} = \frac{\pi^4}{30\, \zeta(3)} \approx 3.896976153 \;\; \text{bits/photon.}$$

- This applies to:
  - Burning a lump of coal... (definitely)...
  - Analogue Hawking radiation... (definitely)...
  - Black hole Hawking radiation,
    (unless one blindly extrapolates general relativity up to the Planck scale, and uses non-empirical non-evidence to assert the existence of (strict) event horizons, *aka* absolute horizons)...
- Event horizons are simply not (empirical) physics...

- *Non black-hole applications completely non-controversial*...
- Entropy is hiding in the correlations...
- See again:
  "On burning a lump of coal",
  Ana Alonso-Serrano and Matt Visser,
  Phys. Lett. B **757** (2016) 383
  doi:10.1016/j.physletb.2016.04.023 [arXiv:1511.01162 [gr-qc]].
- *Black-hole applications rather controversial*...
- *(But they should not be controversial)...*

Information puzzle:

- We do have empirical evidence for trapping/apparent horizons in astrophysical black holes...
- For example:
  - ISCOs... ($r \sim 6m$; unstable timelike orbit)...
  - ringdown... (lowest QNMs; $r \sim 3m$; unstable null orbit)...
  - ADAFs? advection dominated accretion flows? ($r \sim 2m$)...
  - non-echoes... ($r \sim 2m$)...
- We do not have,
  (and in a very precise technical sense, we cannot ever have),
  empirical evidence for *event* horizons in astrophysical black holes...
- No event horizon, no (intrinsic) information puzzle...
- No event horizon, still desirable to calculate entropy fluxes...
- Event horizons are simply not (empirical) physics...

Information puzzle:

- Non-empirical theory verification can easily lead one to into a scientific wasteland of uncontrolled speculation...
- Simple test-case for non-empirical theory verification:
  Carefully analyze the difference between event horizons *versus* apparent/trapping horizons...
- The information puzzle becomes a "problem", (not even a paradox), only if one indulges in an extended bout of non-empirical theory extrapolation...
- But event horizons are simply not (empirical) physics...

String integers?

Words matter...

- One of my pet peeves: "*String integers*".
- Some members of the string community use the word "*integer*" when they mean "*some parameter which might (or might not) become an integer in the extremal supersymmetric limit*".
- For example:
  Horowitz/Maldacena/Strominger carefully say of the "*integer*" parameters they introduce for counting string black hole microstates:
  "*we will refer to them as the numbers of branes, antibranes and strings because (as will be seen) they reduce to those numbers in certain limits where these concepts are well defined*".
  [Physics Letters **B383** (1996) 151-159, hep-th/9603109]
- This careful qualification by Horowitz/Maldacena/Strominger is then *often lost in the subsequent literature*.

String integers?

Words matter...

- You will often see claims to the effect that string theory implies a quantization of outer horizon areas

$$A_+ = 8\pi L_P^2 \left\{ \sqrt{N_1} + \sqrt{N_2} \right\}; \qquad N_1, N_2 \in \mathbb{N}.$$

- In situations where there is both an inner (Cauchy) horizon and outer (event) horizon one often encounters the stronger claim that

$$A_+ A_- = (8\pi L_P^2)^2 \; N; \qquad N \in \mathbb{N}.$$

- This would imply

$$A_\pm = 8\pi L_P^2 \left\{ \sqrt{N_1} \pm \sqrt{N_2} \right\}; \qquad N_1, N_2 \in \mathbb{N}.$$

- Note the loss of qualifying comments regarding the "*integers*" N_i.
- *These unqualified claims are simply wrong*...

String integers?

Words matter...

- For a Kerr–Newman black hole
  (mass m, charge Q, angular momentum $J = ma$)

$$A_\pm = 4\pi(r_\pm^2 + a^2) = 4\pi \left\{ 2m^2 - Q^2 \pm 2m\sqrt{m^2 - a^2 - Q^2} \right\}.$$

- Then (theoretician's units)

$$A_+ A_- = (8\pi)^2 \left[J^2 + \frac{Q^4}{4} \right].$$

- Then (SI units)

$$A_+ A_- = (8\pi L_P^2)^2 \left[j(j+1) + \frac{\alpha^2 q^4}{4} \right]; \qquad j \in \mathbb{N}/2; \quad q \in \mathbb{Z}.$$

- *Can you see a problem here?*

String integers?

Words matter...

- If you accept the (unqualified) string theoretic claims regarding area quantization then

$$\left[j(j+1) + \frac{\alpha^2 q^4}{4} \right] = N; \qquad j \in \mathbb{N}/2; \quad q \in \mathbb{Z}; \quad N \in \mathbb{N}.$$

- That is — string theory has made a "*prediction*"...

$$\alpha = 2\sqrt{m}; \qquad m \in \mathbb{N}.$$

- *The fine structure constant is an $\sqrt{\text{integer}}$ multiple of 2!*
- This is in gross conflict with empirical reality...
- *Significant evasive redefinition of terms required*...

Words matter...

String integers?

Words matter...

- *Significant evasive redefinition of terms required*...
- "Integers" $\Longrightarrow$ "string integers"
  $\Longrightarrow$ "extremal supersymmetric limit"...
- "Integers" $\Longrightarrow$ "charges"...
- "Universal" $\Longrightarrow$ "an effective low energy description of black holes"...
- (Not really fixing the problem...)
- *Referee*:
  "*There are a few places in the literature that have tried to generalize these [area quantization] results too simplistically, and this paper might be a useful antidote.*"
- (Few places? Many places... Many very bold claims...)

Words matter...

String integers?

See the discussion in:

- "*Quantization of area for event and Cauchy horizons of the Kerr-Newman black hole*",
  Matt Visser
  JHEP **1206** (2012) 023
  doi:10.1007/JHEP06(2012)023
  [arXiv:1204.3138 [gr-qc]].

Event horizon telescope?

Event horizon telescope?

- Another one of my pet peeves:
  "*The event horizon telescope*".
- There is simply no way any astronomer, ever,
  will "*resolve the event horizon*".
- This is simply a logical impossibility.
- With enough work on highly spinning Kerr black holes they might get somewhat close to the apparent/trapping horizon...
- "*The near-horizon telescope?*"...
  (but even that requires some lucky accidents...)
- Once you check what they are actually doing, best to call it:
  "*The light-ring telescope*"....
- That is scientifically honest...

Summary

Summary:

- Entropy rises, except when it doesn't...
- Entropy can be both context dependent and observer dependent...
- Coarse-graining can be physical and irreversible;
  or a reversible gedanken-process...
- *Coarse-graining* is *nowhere near as well-understood* as people would like to think...
- Central to understanding the Hawking evaporation of black holes...
- Lots of tricky and subtle mathematics and physics involved...

Summary:

- There is a crucial difference between the "qualitative" and "quantitative" information loss problems.
  - The "qualitative" problem is this:
    If a spacelike singularity forms (in the strict mathematical sense), then there will be a (strict mathematical) event horizon, and unavoidably some loss of unitarity associated with any matter that might cross the event horizon.
  - The "quantitative" problem is this:
    How much information is lost behind the event horizon, (if it forms), and how much comes out in the Hawking radiation?
- Some extremely interesting matters of principle to consider...

End: