# The Kosterlitz-Thouless transition


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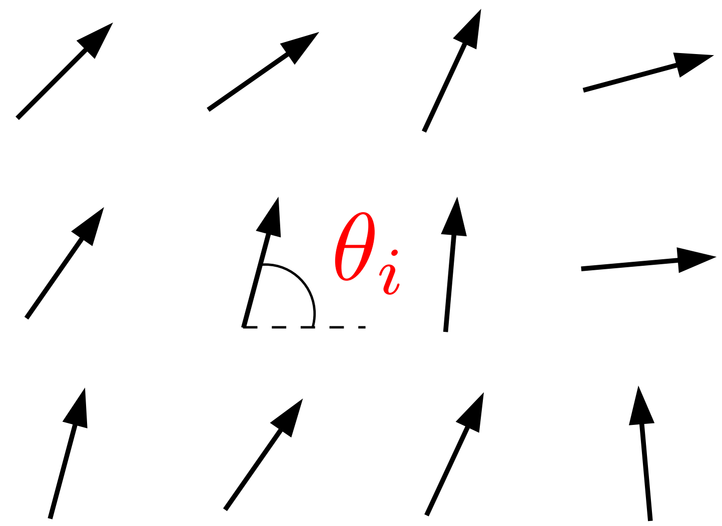
# The XY model

Model for a 2D easy-plane magnet




$$H_{XY} = -\tilde{J} \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$

Continuum limit

$$H_{XY} = \frac{J}{2} \int d^2 r \left( \vec{\nabla} \theta(\vec{r}) \right)^2$$

No ordered state at any finite temperature due to spin-waves!

Mermin and Wagner 1966

# The XY model continued

Vortices

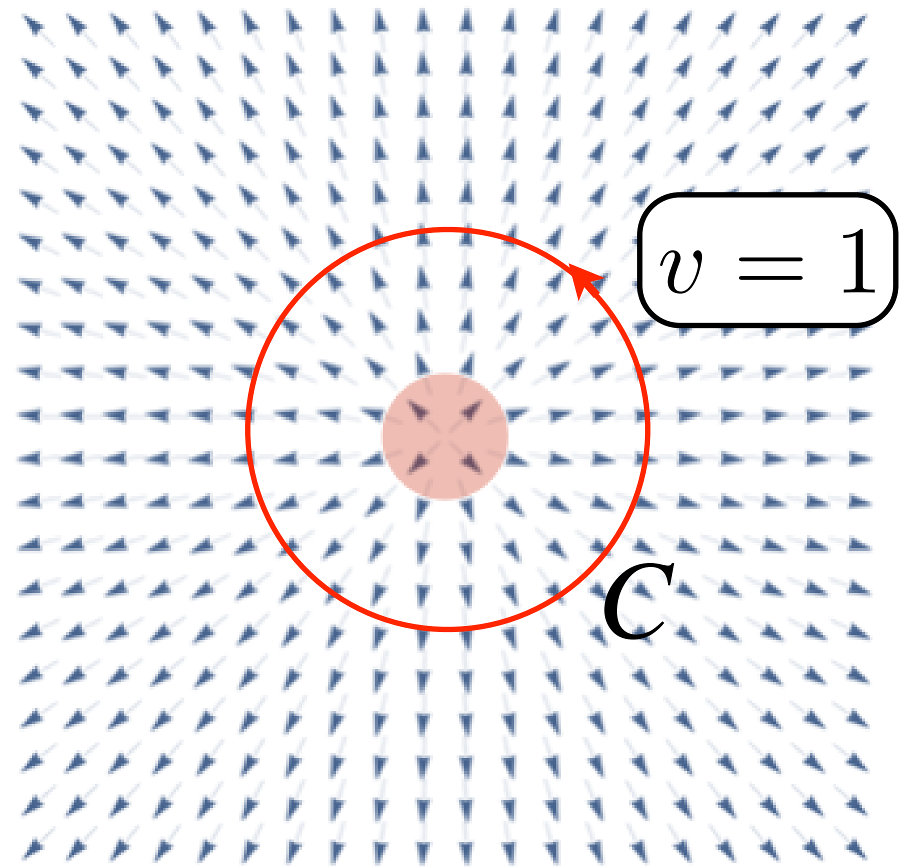
Topological objects!

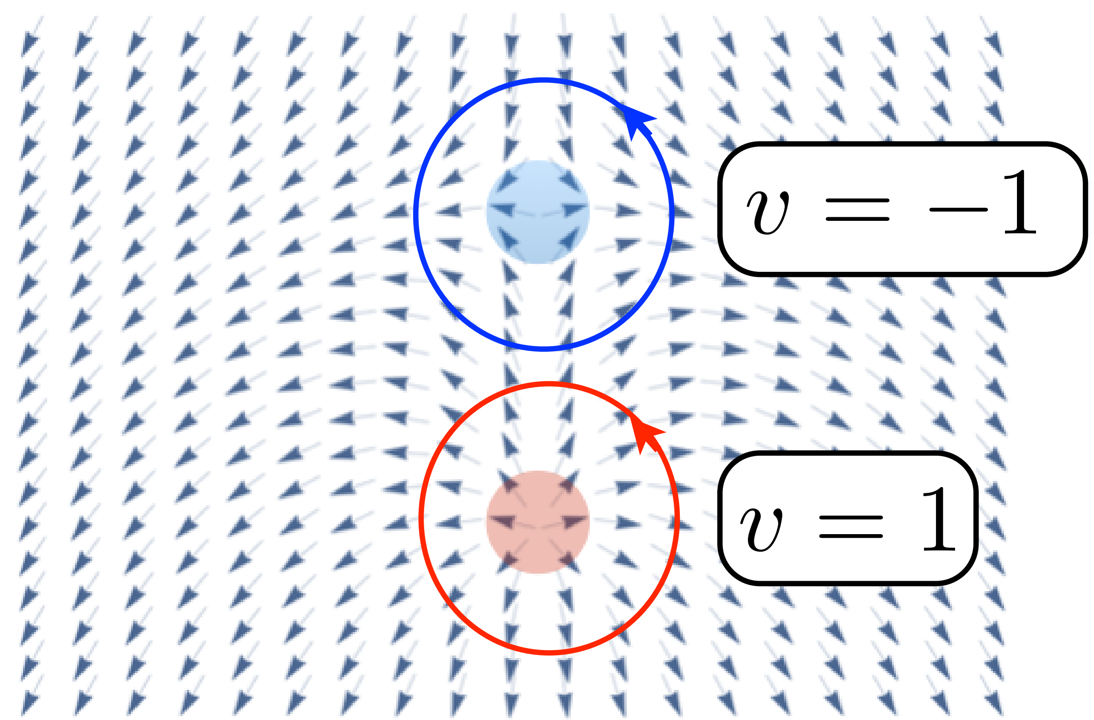
cannot be transformed into a state with all spin aligned by a continuos rotation

$$\vec{S}'(\vec{r}) = \mathcal{R}(\vec{r})\vec{S}(\vec{r})$$

Vorticity: $$v = \frac{1}{2\pi}\oint_C d\vec{r}\cdot\vec{\nabla}\theta(\vec{r})$$

measures rotation (in units of $2\pi$) of the spin vector along the curve



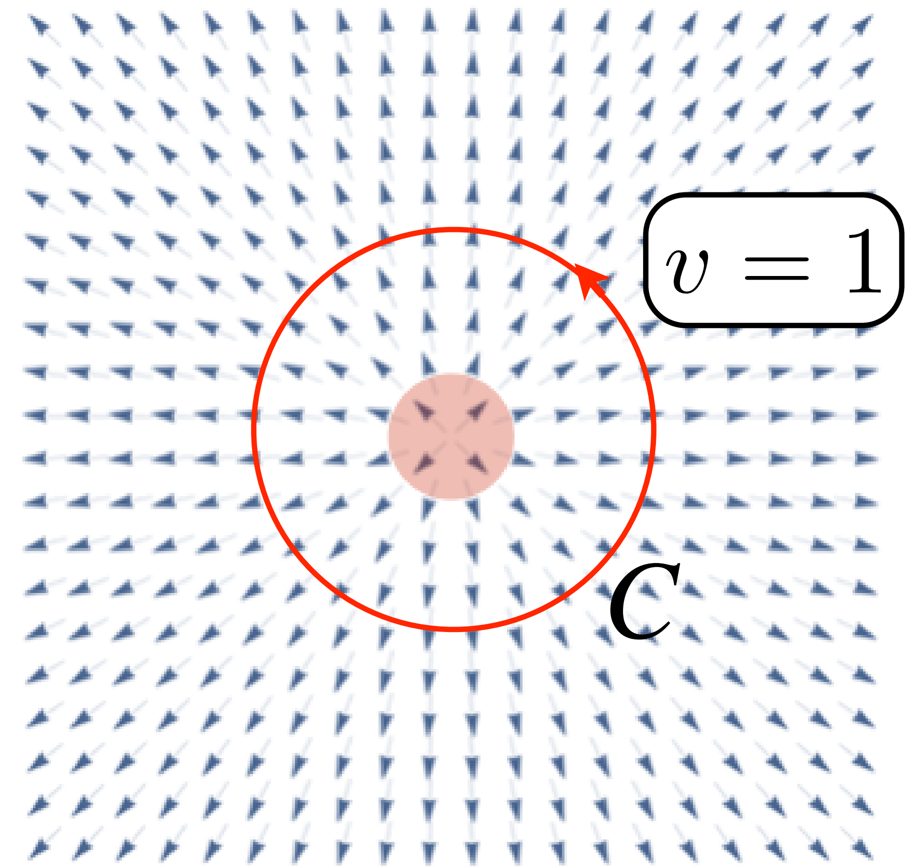




Scientific background, Royal Swedish Academy of Sciences

http://www.nobelprize.org

# XY model continued

$$H_{XY} = \frac{J}{2}\int d^2r\left(\vec{\nabla}\theta(\vec{r})\right)^2$$




Energy cost of a vortex

consider rotationally symmetric vortex with

$v = \pm 1 \longrightarrow |\vec{\nabla}\theta(\vec{r})| = 1/r$

system size

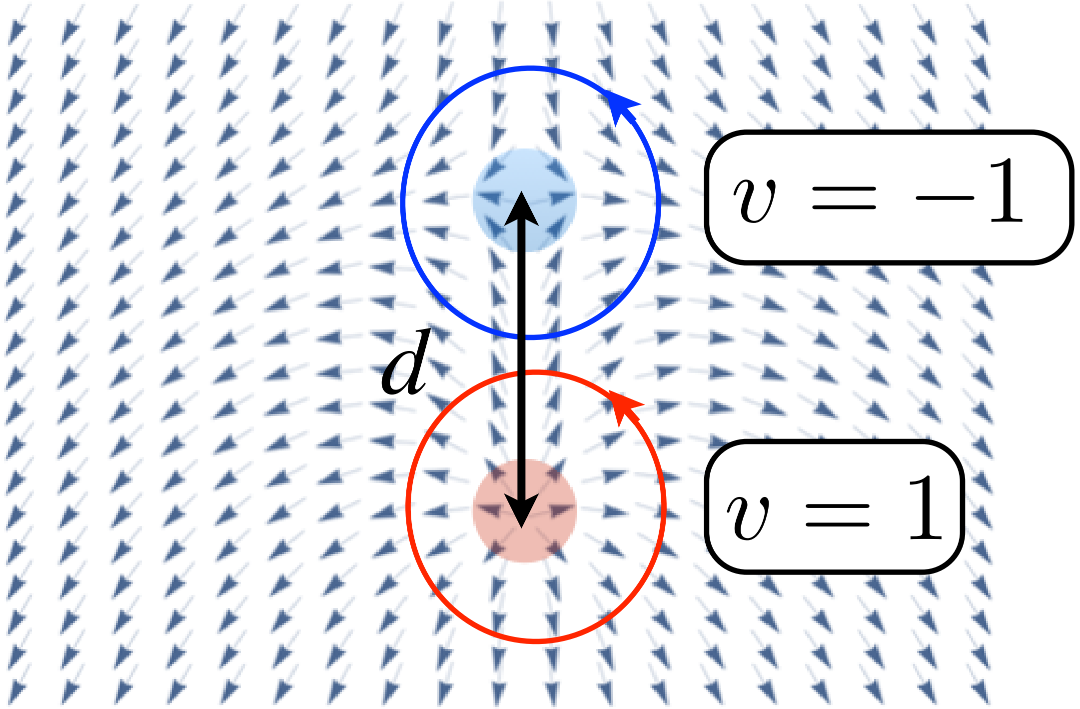
$$E_v = \frac{J}{2}\int d^2r\,\frac{1}{r^2} = J\pi\ln\frac{L}{a}$$

vortex core size

large energy scale!!

# XY model continued

$$H_{XY} = \frac{J}{2} \int d^2 r \left( \vec{\nabla} \theta(\vec{r}) \right)^2$$




Energy cost of a vortex-antivortex pair

distance between the vortices

$$E_{\rm p} = J 2\pi \ln \frac{d}{a}$$

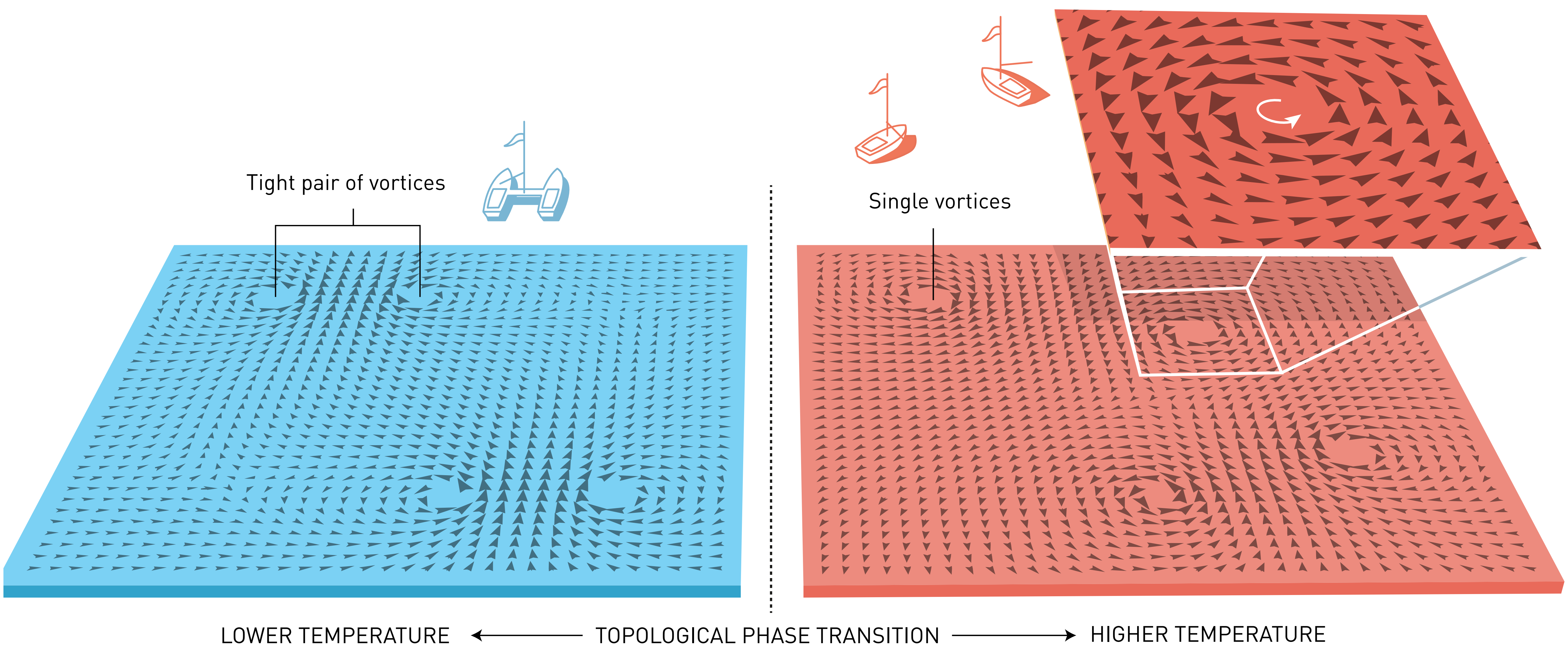
vortex core size

v-av pairs can be thermally excited!

# Kosterlitz-Thouless transition

gas of v-av pairs → unbound vortices

low $T$ — high $T$

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# Kosterlitz-Thouless transition

gas of v-av pairs → unbound vortices

low $T$ → high $T$

Free-energy for a single vortex

$$F = E - TS = \underbrace{J\pi \ln\left(\frac{L}{a}\right)}_{\text{dominates at small } T} - \underbrace{Tk_B \ln\left(\frac{L^2}{a^2}\right)}_{\text{dominate at large } T}$$

Critical temperature:

$$\boxed{T_{\mathrm{KT}} = \frac{J\pi}{2k_B}}$$

Topological phase transition!

J M Kosterlitz and D J Thouless, J. Phys C: Solid State Phys. **6**, 1181 (1973).

# Kosterlitz-Thouless transition

gas of v-av pairs ⟶ unbound vortices

low $T$ high $T$

Topological phase transition!

Physical systems

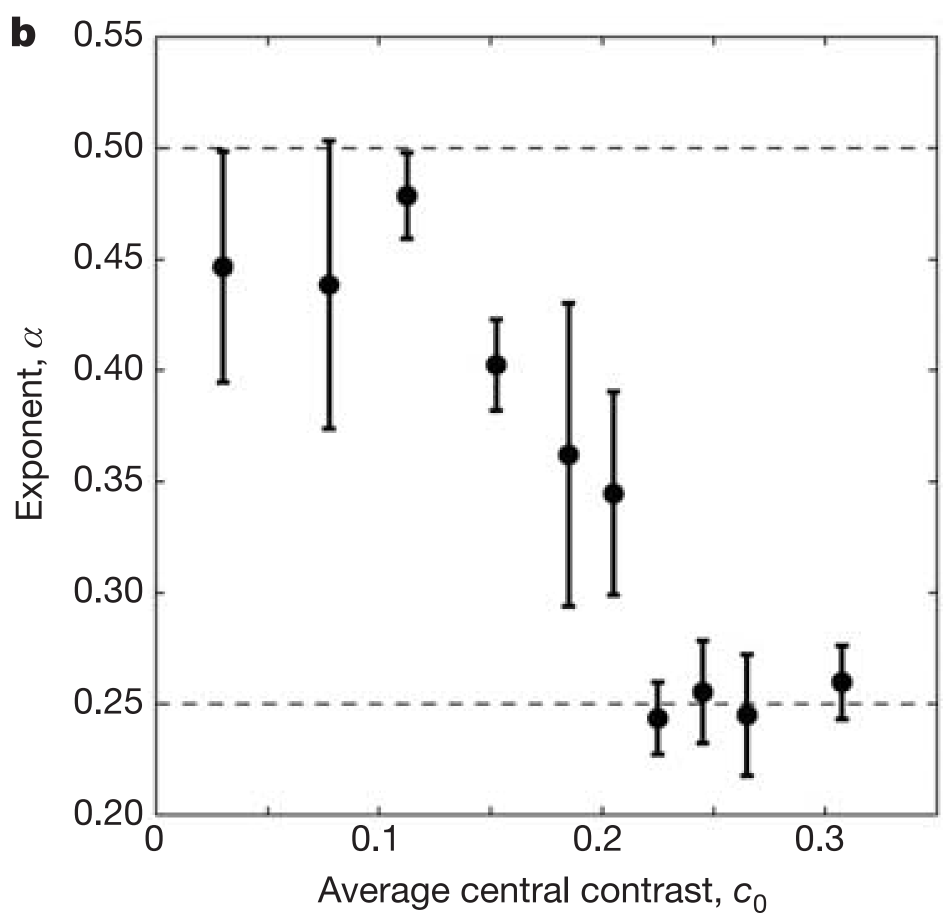
- thin films of superfluid $^4$He
- disordered superconducting thin films
- planar arrays of Josephson junctions
- melting of 2D solids

Vol 441|29 June 2006|**doi:10.1038/nature04851**

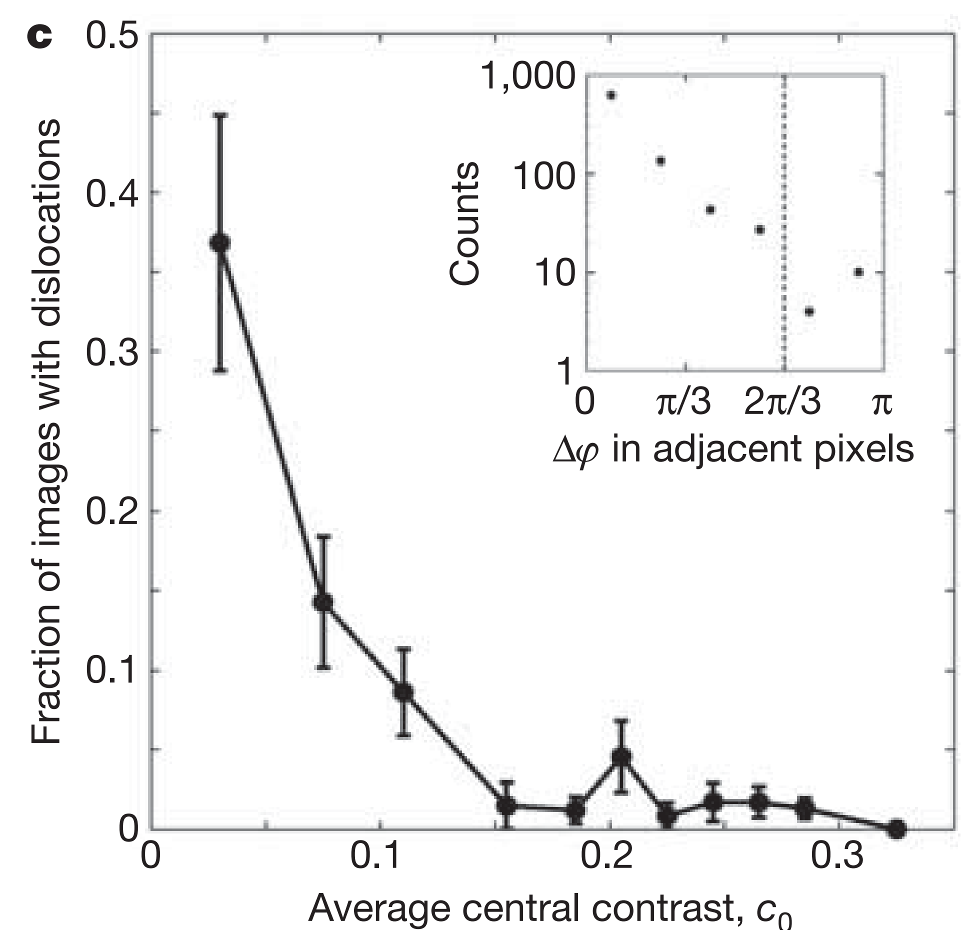
LETTERS

# Berezinskii–Kosterlitz–Thouless crossover in a trapped atomic gas

Zoran Hadzibabic[1], Peter Krüger[1], Marc Cheneau[1], Baptiste Battelier[1] & Jean Dalibard[1]




hight $T$ ← low $T$




hight $T$ ← low $T$