Victoria University of Wellington

Te Whare Wānanga o te Ūpoko o te Ika a Maui

Lecture 2 of 4: Lorentz invariance and the zero-point stress-energy

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Abstract:

- Some 65 years ago (1951) Wolfgang Pauli noted that the zero-point energy density could be set to zero by a carefully fine-tuned cancellation between bosons and fermions.
- In this lecture I will argue in a slightly different direction: The zero-point energy density is only one component of the zero-point stress energy tensor, and it is this tensor quantity that is in many ways the more fundamental object of interest.
- I shall demonstrate that Lorentz invariance of the zero-point stress energy tensor implies finiteness of the zero-point stress energy tensor, and vice versa.

— arXiv:1610.07264 [gr-qc] —

Outline:

Introduction

Introduction:

- Remember:
- We are interested in making sense of the zero-point energy density:

$$\rho_{zpe} = \frac{\hbar}{2} \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\, \omega(\vec{k}).$$

- Naively divergent...

Pauli 1951

Pauli 1951

- Very definitely dead-tree technology...
- Not online anywhere...
- 1971 translation of 1951 lectures at ETH Zurich...

Pauli 1951: Zero-point energy density...

- Pauli was worried about the zero-point energy density:

$$\rho_{zpe} = \frac{\hbar}{2} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \sqrt{m^2 + k^2} ...$$

- Flat space, infinite volume, free-field calculation...
- No interactions as yet...
- (Yes this argument will [sometimes] survive interactions...)
- Naively this diverges to infinity...
- What to do?

The question:

One can ask whether these zero-point energies can compensate one another. We have

$$\begin{cases} \text{spin } 0: & \dfrac{E_0}{V} = \left(\dfrac{1}{2\pi}\right)^3 \displaystyle\int \sqrt{k^2+m^2}\, \mathrm{d}^3k; \\ \text{spin } \tfrac{1}{2}: & \dfrac{E_0}{V} = -2\left(\dfrac{1}{2\pi}\right)^3 \displaystyle\int \sqrt{k^2+m^2}\, \mathrm{d}^3k; \end{cases}$$

where the masses are, in general, different. For compensation we must calculate

$$\int_0^K k^2\sqrt{k^2+m^2}\,\mathrm{d}k = \frac{K^4}{4} + \frac{1}{4}m^2K^2 - \frac{m^4}{4}\log\frac{2K}{m} + O\left(\frac{1}{K}\right).$$

The answer:

We see that the compensation requirements are

$$\begin{pmatrix}\text{the number of kinds of}\\ \text{spin-zero particles}\end{pmatrix} = 2\times\begin{pmatrix}\text{the number of kinds}\\ \text{of spin-}\tfrac{1}{2}\text{ particles}\end{pmatrix};$$

i.e.,

$$Z_0 = 2\, Z_{\frac{1}{2}}\ .$$

Furthermore,

$$\sum_i (m_0^i)^2 = 2\sum (m_{\frac{1}{2}}^i)^2\,,$$

$$\sum_i (m_0^i)^4 = 2\sum (m_{\frac{1}{2}}^i)^4\,,$$

$$\sum (m_0^i)^4 \log m_0^i = 2\sum_i (m_{\frac{1}{2}}^i)^4 \log m_{\frac{1}{2}}^i\ .$$

These requirements are so extensive that it is rather improbable that they are satisfied in reality.

More modern notation

More modern notation:

$$\rho_{zpe} = \sum_n \left\{ (-1)^{2S_n} g_n \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \, \frac{1}{2} \, \hbar\omega_n(k) \right\}.$$

- Integrate the zero-point energy $\pm\frac{1}{2}\hbar\omega(k)$ over all modes.
- Boson contributions positive, fermion contributions negative.
- Degeneracy factor g includes spin factor $g = 2S + 1$ for massive particles, whereas spin factor is $g = 2$ for massless particles.
- Degeneracy factor g also includes a factor of 2 when particle and antiparticle are distinct, and a factor of 3 due to colour.
- Finally one sums over all particle species indexed by n.
- Sum over the entire particle physics spectrum is Pauli's key insight.

More modern notation:

Now explicitly introduce particle masses:

$$\rho_{zpe} = \sum_n \left\{ (-1)^{2S_n} g_n \, \frac{1}{2} \hbar \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \sqrt{m_n^2 + k^2} \right\}.$$

More modern notation:

Key integral:

$$\begin{aligned}
\int_0^K \mathrm{d}^3 k \, \sqrt{m^2+k^2} &= 4\pi \int_0^K \mathrm{d}k \; k^2 \, \sqrt{m^2+k^2} \\
&= \pi \left\{ K(m^2+K^2)^{3/2} - \frac{1}{2} m^2 K \sqrt{m^2+K^2} \right. \\
&\qquad \left. - \frac{1}{2} m^4 \ln\left(\frac{K+\sqrt{m^2+K^2}}{m} \right) \right\} \\
&= \pi \left\{ K^4 + m^2 K^2 + \frac{m^4}{8} - \frac{1}{2} m^4 \ln(2K/m) \right\} \\
&\qquad + \mathcal{O}\left(\frac{1}{K^2} \right).
\end{aligned}$$

Pauli 1951: Zero-point energy density...

More modern notation:

The net zero-point energy is zero if and only if one imposes three polynomial-in-mass conditions

$$\sum_n (-1)^{2S_n} g_n = 0; \qquad \sum_n (-1)^{2S_n} g_n \; m_n^2 = 0; \qquad \sum_n (-1)^{2S_n} g_n \; m_n^4 = 0;$$

and additionally imposes a fourth logarithmic-in-mass condition

$$\sum_n (-1)^{2S_n} g_n \; m_n^4 \; \ln(m_n^2/\mu^2) = 0.$$

Pauli 1951: Zero-point energy density...

More modern viewpoint:

The net zero-point energy is finite if and only if one imposes the three polynomial-in-mass conditions

$$\sum_n (-1)^{2S_n} g_n = 0; \qquad \sum_n (-1)^{2S_n} g_n \; m_n^2 = 0; \qquad \sum_n (-1)^{2S_n} g_n \; m_n^4 = 0.$$

If this is done, then

$$\rho_{zpe} = \frac{\hbar}{64\pi^2} \sum_n (-1)^{2S_n} g_n \; m_n^4 \; \ln(m_n^2/\mu^2)$$

is finite.

Zero-point stress-energy tensor

Zero-point stress-energy tensor

Zero-point stress-energy tensor:

$$(T_{zpe})^{ab} = \sum_n \left\{ (-1)^{2S_n} g_n \int \frac{\mathrm{d}^3 k}{2\omega_n(k)\;(2\pi)^3}\; \hbar\; k_n^a\, k_n^b \right\}.$$

4-momenta:

$$k^a = (\omega(k); k^i) = \left(\sqrt{m^2+k^2}; k^i\right)$$

Mode contribution:

$$\frac{1}{2}\;\hbar\; k^a k^b$$

Lorentz invariant phase space:

$$\mathrm{d}(LIPS) = \frac{1}{(2\pi)^3}\;\frac{\mathrm{d}^3 k}{(2\omega)}.$$

Zero-point stress-energy tensor:

$$(T_{zpe})^{ab} = \sum_n \left\{ (-1)^{2S_n} g_n \int \frac{\mathrm{d}^3 k}{2\omega_n(k)\,(2\pi)^3}\;\hbar\;\left[\begin{array}{c|c} \omega_n(k)^2 & \omega_n(k)\,k^j \\ \hline \omega_n(k)\,k^i & k^i k^j \end{array}\right]^{ab} \right\}.$$

Rotational invariance:

$$(T_{zpe})^{ab} = \sum_n \left\{ (-1)^{2S_n} g_n \int \frac{\mathrm{d}^3 k}{2\omega_n(k)\,(2\pi)^3}\;\hbar\;\left[\begin{array}{c|c} \omega_n(k)^2 & 0 \\ \hline 0 & \frac{1}{3} k^2\,\delta^{ij} \end{array}\right]^{ab} \right\}.$$

Therefore:

$$(T_{zpe})^{ab} = \left[\begin{array}{c|c} \rho_{zpe} & 0 \\ \hline 0 & p_{zpe}\,\delta^{ij} \end{array}\right].$$

Zero-point stress-energy tensor

Explicitly:

$$(T_{zpe})^{ab} = \left[\begin{array}{c|c} \rho_{zpe} & 0 \\ \hline 0 & p_{zpe}\,\delta^{ij} \end{array}\right],$$

with

$$\rho_{zpe} = \sum_n \left\{(-1)^{2S_n} g_n \,\frac{1}{2}\hbar \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \sqrt{m_n^2 + k^2}\right\},$$

and

$$p_{zpe} = \sum_n \left\{(-1)^{2S_n} g_n \,\hbar \int \frac{\mathrm{d}^3 k}{(2\pi)^3\, 2\sqrt{m_n^2 + k^2}} \,\frac{k^2}{3}\right\}.$$

Note:

- Formulae similar to

$$\rho_{zpe} = \pm \frac{\hbar}{2} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \sqrt{m^2 + k^2};$$

$$p_{zpe} = \pm \frac{\hbar}{6} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{k^2}{\sqrt{m^2 + k^2}};$$

are not uncommon...

- Zero-point pressure is as important as zero-point energy...

Lorentz invariance

Lorentz invariance requires:

$$\rho_{zpe} + p_{zpe} = 0.$$

That is:

$$\rho_{zpe} + p_{zpe} = \sum_n \left\{ (-1)^{2S_n} g_n \, \hbar \int \frac{\mathrm{d}^3 k}{2\sqrt{m_n^2 + k^2}(2\pi)^3} \, \left(m_n^2 + \frac{4}{3} k^2 \right) \right\} = 0.$$

This is an extremely powerful constraint...

Key integral:

$$\int_0^K \frac{\mathrm{d}^3 k}{\sqrt{m^2+k^2}} \left(m^2 + \frac{4}{3}k^2\right) = 4\pi \int_0^K \frac{\mathrm{d}k}{\sqrt{m^2+k^2}} \left(k^2 m^2 + \frac{4}{3}k^4\right)$$

$$= \frac{4\pi}{3} K^3 \sqrt{K^2+m^2}$$

$$= \frac{\pi}{6}\left(8K^4 + 4m^2 K^2 - m^4\right) + \mathcal{O}\left(\frac{1}{K^2}\right).$$

Consequently:

(Lorentz invariance) $\iff$ (Pauli's three polynomial-in-mass constraints)

That is:

(Lorentz invariance) $\iff$

$$\sum_n (-1)^{2S_n} g_n = 0; \qquad \sum_n (-1)^{2S_n} g_n\; m_n^2 = 0; \qquad \sum_n (-1)^{2S_n} g_n\; m_n^4 = 0.$$

Consequently:

$$(\text{Lorentz invariance}) \iff (\text{zero-point energy-density is finite})$$

Explicitly:

$$\rho_{zpe} = -p_{zpe} = \frac{\hbar}{64\pi^2} \sum_n (-1)^{2S_n} g_n \; m_n^4 \; \ln(m_n^2/\mu^2).$$

Where automatically:

$$\sum_n (-1)^{2S_n} g_n = 0; \qquad \sum_n (-1)^{2S_n} g_n \; m_n^2 = 0; \qquad \sum_n (-1)^{2S_n} g_n \; m_n^4 = 0.$$

Implications?

Implications?

This analysis impacts on a number of wider issues:

- Beyond standard model (BSM) physics.
- Naive estimates of the cosmological constant.
- Supersymmetry (being neither necessary nor sufficient).
- Sakharov-style induced gravity.

BSM physics?

BSM physics?

- Split into SM & BSM sectors:

$$\sum_{BSM} (-1)^{2S_n} g_n = -\sum_{SM} (-1)^{2S_n} g_n;$$

$$\sum_{BSM} (-1)^{2S_n} g_n \; m_n^2 = -\sum_{SM} (-1)^{2S_n} g_n \; m_n^2;$$

$$\sum_{BSM} (-1)^{2S_n} g_n \; m_n^4 = -\sum_{SM} (-1)^{2S_n} g_n \; m_n^4.$$

- There must be BSM physics...

Cosmological constant?

Common assertion:

$$\rho_{cc} = \rho_{zpe} = -p_{zpe}.$$

Define mass scales:

$$\sum_{SM} (-1)^{2S_n} g_n \; m_n^4 \; \ln(m_n^2/\mu_{SM}^2) = 0;$$

$$\sum_{BSM} (-1)^{2S_n} g_n \; m_n^4 \; \ln(m_n^2/\mu_{BSM}^2) = 0.$$

Calculate ρ_{cc} in terms of these mass scales...

$$
\begin{aligned}
\rho_{cc} &= \rho_{zpe} = -p_{zpe} \\
&= \frac{\hbar}{64\pi^2} \sum_n (-1)^{2S_n} g_n \; m_n^4 \; \ln(m_n^2/\mu^2) \\
&= \frac{\hbar}{64\pi^2} \sum_n (-1)^{2S_n} g_n \; m_n^4 \; \ln(m_n^2/\mu_{BSM}^2) \\
&= \frac{\hbar}{64\pi^2} \sum_{SM} (-1)^{2S_n} g_n \; m_n^4 \; \ln(m_n^2/\mu_{BSM}^2) \\
&\qquad + \frac{\hbar}{64\pi^2} \sum_{BSM} (-1)^{2S_n} g_n \; m_n^4 \; \ln(m_n^2/\mu_{BSM}^2) \\
&= \frac{\hbar}{64\pi^2} \sum_{SM} (-1)^{2S_n} g_n \; m_n^4 \; \ln(m_n^2/\mu_{BSM}^2)
\end{aligned}
$$

Cosmological constant?

$$
\begin{aligned}
\rho_{cc} &= \rho_{zpe} = -p_{zpe} \\
&= \frac{\hbar}{64\pi^2} \sum_{SM} (-1)^{2S_n} g_n \; m_n^4 \; \ln(m_n^2/\mu_{BSM}^2) \\
&= \frac{\hbar}{64\pi^2} \sum_{SM} (-1)^{2S_n} g_n \; m_n^4 \; \ln(m_n^2/\mu_{SM}^2) \\
&\qquad + \frac{\hbar}{64\pi^2} \sum_{SM} (-1)^{2S_n} g_n \; m_n^4 \; \ln(\mu_{SM}^2/\mu_{BSM}^2) \\
&= \frac{\hbar}{64\pi^2} \sum_{SM} (-1)^{2S_n} g_n \; m_n^4 \; \ln(\mu_{SM}^2/\mu_{BSM}^2).
\end{aligned}
$$

Cosmological constant?

That is:

$$\rho_{cc} = \rho_{zpe} = -p_{zpe} = -\frac{\hbar}{64\pi^2} \left\{ \sum_{SM} (-1)^{2S_n} g_n \, m_n^4 \right\} \ln\left(\frac{\mu_{BSM}^2}{\mu_{SM}^2} \right).$$

- Here μ_{BSM} is the only unknown...

Supersymmetry:
Neither necessary
nor sufficient

- Supersymmetry is not necessary in order to set up and understand any of the preceding analysis.
- Pauli's 1951 lectures pre-date even the earliest versions of supersymmetry by some 20 years.
- Zumino certainly knew of, partially inspired by, Pauli's result...
- Unbroken supersymmetry automatically satisfies Pauli's constraints...
- Unbroken supersymmetry also in violent conflict with empirical reality.
- Broken supersymmetry, (spontaneously broken or explicitly broken), need not satisfy the second and third (m^2 or m^4) Pauli constraints.
- (The first Pauli constraint will survive supersymmetry breaking.)

- If desired one can rewrite the sum over the particle spectrum in Pauli's constraints as a “supertrace”,

$$\sum_n (-1)^{2S_n} \, g_n \, X_n = \mathrm{Str}[X].$$

- But this is merely a book-keeping device...
- This is not an appeal to supersymmetry...
- Supersymmetry, or lack thereof, is at best logically orthogonal to the questions addressed in this seminar.

Sakharov-style induced gravity

- The analysis so far is strictly a flat-space Minkowski result...
- But due to the locally Euclidean nature of spacetime it will still govern the dominant short-distance physics in curved spacetime.
- There will certainly be sub-leading curvature-dependent terms — which are more easily dealt with by a short-distance asymptotic expansion of the heat kernel in terms of Seeley–DeWitt coefficients.
- This naturally leads to the concept of Sakharov-like induced gravity.
- The present analysis could easily be modified and extended to further elucidate the induced gravity scenario.
- More on this in Lecture 4.

- Care must be taken to add and subtract only finite regulated physically meaningful quantities, before sending the regulator to infinity.
- Over-enthusiastic application of curved space (or even flat space) renormalization techniques can easily eliminate the interesting parts of the physics.

Some numbers

Some numbers:

Pauli's three sum rules can be written as:

$$\sum_{BSM} (-1)^{2S_n} g_n = N_{BSM} = -\sum_{SM} (-1)^{2S_n} g_n;$$

$$\sum_{BSM} (-1)^{2S_n} g_n\; m_n^2 = (M_2)^2 = -\sum_{SM} (-1)^{2S_n} g_n\; m_n^2;$$

$$\sum_{BSM} (-1)^{2S_n} g_n\; m_n^4 = (M_4)^4 = -\sum_{SM} (-1)^{2S_n} g_n\; m_n^4.$$

Pauli's three sum rules can be written as (book-keeping only):

$$\mathrm{Str}_{BSM}[1] = N_{BSM} = -\mathrm{Str}_{SM}[1];$$

$$\mathrm{Str}_{BSM}[m^2] = (M_2)^2 = -\mathrm{Str}_{SM}[m^2];$$

$$\mathrm{Str}_{BSM}[m^4] = (M_4)^4 = -\mathrm{Str}_{SM}[m^4].$$

Set:

$$d = (-1)^{2S} g$$

Some numbers:

Table: Calculations of $\mathrm{Str}_{SM}[1]$, $\mathrm{Str}_{SM}[\hat{m}^2]$, $\mathrm{Str}_{SM}[\hat{m}^4]$, and $\mathrm{Str}_{SM}[\hat{m}^4 \ln \hat{m}^2]$, working in the SM sector after spontaneous electro-weak symmetry breaking.

particle	d	mass/GeV	$\hat{m} = m/m_H$	$d \times \hat{m}^2$	$d \times \hat{m}^4$	$d \times \hat{m}^4 \ln(\hat{m}^2)$
Higgs	1	125.02	1	1	1	0
Z^0	3	91.1876	0.729384099	1.59600349	0.849075713	-0.535859836
$W^\pm$	6	80.385	0.642977124	2.480517489	1.025494502	-0.905811363
top	-12	173.21	1.385458327	-23.0339373	-44.21352229	-28.82995836
bottom	-12	4.66	0.037274036	-0.016672245	-2.31636E-05	0.000152392
charm	-12	1.27	0.010158375	-0.001238311	-1.27784E-07	1.17292E-06
strange	-12	0.096	0.000767877	-7.07562E-06	-4.17204E-12	5.98427E-11
up	-12	0.0022	1.75972E-05	-3.71593E-09	-1.15068E-18	2.51947E-17
down	-12	0.0047	0.000037594	-1.69597E-08	-2.39693E-17	4.8843E-16
gluons	16	0	0	0	0	0
tau	-4	1.77686	0.014212606	-0.000807993	-1.63213E-07	1.38849E-06
muon	-4	0.105658375	0.000845132	-2.85699E-06	-2.0406E-12	2.88786E-11
electron	-4	0.000510999	4.08734E-06	-6.68253E-11	-1.11641E-21	2.77039E-20
neutrinos	-12	0.000000002	1.59974E-11	-3.07102E-21	-7.85929E-43	3.90742E-41
photon	2	0	0	0	0	0
$\mathrm{Str}_{SM}[X]$	-68	—	—	-17.97614482	-41.33897553	-30.27147461

- Within the SM sector, the three quantities $\mathrm{Str}_{SM}[\hat{m}^2]$, $\mathrm{Str}_{SM}[\hat{m}^4]$, and $\mathrm{Str}_{SM}[\hat{m}^4 \ln \hat{m}^2]$ are utterly dominated by the top quark — with the top quark accounting for some 80% to 95% of the SM effect.
- This happens for two reasons, first the top quark is simply the heaviest SM particle, and secondly the degeneracy factor for quarks $(g = 12)$ is so high.
- Even if one looks slightly beyond the top quark itself, between them the Higgs, Z^0, $W^\pm$, and top quark are the only particles making any appreciable contribution to these quantities from within the SM sector.

Some numbers:

- Note

$$N_{BSM} = 68; \qquad M_2 = 4.240\ m_H; \qquad M_4 = 2.536\ m_H.$$

- Assuming the Pauli constraints, there are at least 68 bosonic degrees of freedom in the BSM sector.
- The m^2 and m^4 sum rules, which determine M_2 and M_4, indicate that the BSM spectrum is boson dominated.

Some numbers:

Define

$$\mathrm{Str}_{SM}[m^4 \ln(m^2/\mu_{SM}^2)] = 0; \qquad \mathrm{Str}_{BSM}[m^4 \ln(m^2/\mu_{BSM}^2)] = 0.$$

Then

$$\mu_{SM}^2 = m_H^2 \exp\left(\frac{\mathrm{Str}_{SM}[\hat{m}^4 \ln \hat{m}^2]}{\mathrm{Str}_{SM}[\hat{m}^4]}\right) = 2.080\ m_H^2 = (1.442)^2\ m_H^2.$$

Unfortunately we have no similar result for μ_{BSM}...

The BSM spectrum is, (at this stage), not all that tightly constrained.

Some numbers:

The cosmological constant can be estimated by

$$\rho_{cc} = \rho_{zpe} = -p_{zpe} = -\frac{\hbar}{64\pi^2}\left\{\mathrm{Str}_{SM}[m^4]\right\} \ln\left(\frac{\mu_{BSM}^2}{\mu_{SM}^2}\right)$$
$$= 0.06545\ \hbar\ m_H^4\ \ln\left(\frac{\mu_{BSM}^2}{\mu_{SM}^2}\right).$$

- Here μ_{BSM} is the only place that unknown BSM physics now enters into the cosmological constant.
- At least the energy scale for the cosmological constant is not off by the extremely naive factor 10^{123}; it is now more like 10^{55}.
- It is not supersymmetry that is responsible for this reduction; it is the much more basic symmetry of Lorentz invariance for the zero-point stress-energy.

- Observational data regarding the cosmological constant now suggests

$$\ln\left(\frac{\mu_{BSM}^2}{\mu_{SM}^2}\right) \lesssim 10^{-55}.$$

- Rather than being a fine tuning, it is probably best to interpret this as an extremely tight observational (rather than theoretical) constraint on the BSM spectrum.
- Equivalently

$$\mathrm{Str}_{SM}[\hat{m}^4 \ln(\hat{m}^2)] + \mathrm{Str}_{BSM}[\hat{m}^4 \ln(\hat{m}^2)] \lesssim 10^{-55},$$

while each of these terms individually is of order ± 30.

Some numbers:

Table: Str[1], Str[$\hat{m}^2$], Str[$\hat{m}^4$], and Str[$\hat{m}^4 \ln \hat{m}^2$] in the SM sector — — before electro-weak symmetry breaking

particle	d	(mass/GeV)	$\hat{m} = m/m_H$	$d \times \hat{m}^2$	$d \times \hat{m}^4$	$d \times \hat{m}^4 \ln \lvert\hat{m}^2\rvert$
Higgs	4	$(125.02)i$	i	-4	$+4$	0
W	6	0	0	0	0	0
top	-12	0	0	0	0	0
bottom	-12	0	0	0	0	0
charm	-12	0	0	0	0	0
strange	-12	0	0	0	0	0
up	-12	0	0	0	0	0
down	-12	0	0	0	0	0
gluons	16	0	0	0	0	0
$(\text{leptons})_L$	-12	0	0	0	0	0
$(\text{leptons})_R$	-12	0	0	0	0	0
hyper-photon	2	0	0	0	0	0
$\text{Str}_{SM}[X]$	-68	—	—	-4	$+4$	0

Some numbers:

- Note that $\mathrm{Str}_{SM}[1] = -68$ is unchanged, as it should be.
  - Spontaneous symmetry breaking merely moves bosonic and fermionic modes around, it does not create or destroy modes.
  - So $\mathrm{Str}_{BSM}[1] = +68$ as previously.
  - The BSM sector contains at least 68 bosonic degrees of freedom.
- Note that $\mathrm{Str}_{SM}[m^2] = -4m_H^2$ and $\mathrm{Str}_{SM}[m^4] = +4m_H^4$ are both changed compared to the broken phase.
  - This is not unexpected, and actually gives us extremely useful information...
  - Enforcing Pauli's sum rules, this implies that both $\mathrm{Str}_{BSM}[m^2]$ and $\mathrm{Str}_{BSM}[m^4]$ must change during spontaneous symmetry breaking.
  - This in turn implies that at least part of the BSM spectrum must be sensitive to the onset of spontaneous symmetry breaking...
  - So at least part of the BSM spectrum must couple to the Higgs.

Some numbers:

- Before spontaneous symmetry breaking the BSM spectrum satisfies:

$$\mathrm{Str}_{BSM}[1] = +68; \qquad \mathrm{Str}_{BSM}[m^2] = 4m_H^2; \qquad \mathrm{Str}_{BSM}[m^4] = -4m_H^4.$$

- The BSM sector is (still) boson dominated as it should be.
- But now $\mathrm{Str}_{BSM}[m^4] = -4m_H^4$ in the unbroken phase...
- So there must be at least one fermion in the BSM spectrum...
- So at least 69 bosonic degrees of freedom in the BSM sector...
- This is as far as I have been able to push things, (so far), based on these very general principles...

Conclusions

Conclusions:

The key message to extract from this lecture is the central importance of Lorentz invariance in controlling the finiteness of the zero-point stress-energy tensor:

- Lorentz invariance
  $\Longrightarrow$ the three polynomial-in-mass Pauli constraints
  $\Longrightarrow$ finiteness.
- Finiteness
  $\Longrightarrow$ the three polynomial-in-mass Pauli constraints
  $\Longrightarrow$ Lorentz invariance.

This deep and intimate connection between the fundamental physical issues of symmetry and finiteness seems rather oddly to not have previously been developed to the extent that it could.

End of Lecture 2.