

# Inquiry-based learning in mathematics

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- An overview of inquiry-based learning (IBL) in (tertiary) mathematics

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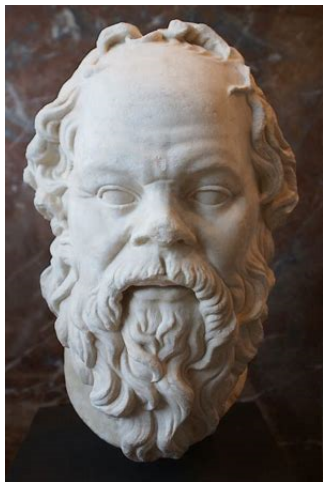
- An overview of inquiry-based learning (IBL) in (tertiary) mathematics
- My experience with IBL
- Practicum for you as IBL learners

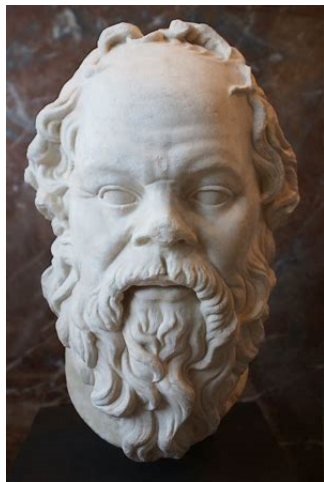
From the *Journal of Inquiry-Based Learning in Mathematics*:

Inquiry-based learning (IBL) refers to any pedagogy that utilizes presentations or student-centered activities to **develop in students the confidence and ability to do mathematics on their own**. Instructors typically supply students with carefully crafted activities or course notes consisting of a sequence of definitions, problems or theorems. Instructors then serve as mentors, listening to the students, reviewing their work, and giving them the minimal information they need to understand the defined concepts, solve the problems, or prove the theorems.

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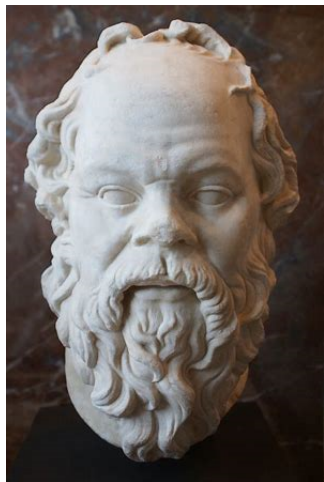
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Socrates. 470–399 BCE

The **Socratic Method** teaches by asking questions:

“What is justice?” *The Republic* by Plato





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What I see, I remember.  
What I do, I understand.



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Xunzi. 300-230 BCE





The student is taught the best who is told the least.



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R.L. Moore. 1882 – 1974

Developed the *Moore Method* which has a big following in the USA







A great discovery solves a great problem, but there is a grain of discovery in the solution of any problem. Your problem may be modest, but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery.



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George Polya. 1887-1985





You learn the most about a subject when you are figuring it out directly for yourself, and learn the least when you are trying to figure out what someone else is saying about it.



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Kenneth P. Bogart. 1943–2005

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You get on the bike and try!

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## Lisa the teacher: the early years

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  - ▶ Give students a series of exercises that allow them to develop the material themselves.

## Sample IBL prob and stats questions:

Some years ago a well-known public official left California and moved to Alabama. A local California reporter revealed both his regional chauvinism and his feelings about the official when he remarked that “on this occasion he raised the mean IQ in both states”.

Explain how this is statistically possible.

## Sample IBL prob and stats questions:

A man was arrested on suspicion of murder. The investigating officer collected all the relevant information, added his impressions of the suspect, and arrived at the conclusion that the man's probability of guilt was 0.60.

As the investigation went on, it was learned (beyond any reasonable doubt) that the murderer's blood type was O. The relative frequency of blood type O in the population is 0.33 (that is the probability that a randomly selected person in the population has blood type O). The suspect's blood was tested and found to be O.

Compute the 'posterior' probability of this suspect's guilt (from the officer's point of view) considering all the data.



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- Develop notes and go full *Moore method*.
- IBL courses can be used achieve outcomes beyond content like:
  - ▶ writing skills
  - ▶ team work
  - ▶ oral communication and presentation skills.

# Practicum for you to be an IBL learner: Voting Theory

A large family is trying to choose where they should go out to dinner. They have narrowed it down to three options:

Argentine steak house (A)

Biergarten (B)

Chinese restaurant (C)

Each family member ranked the options in order of their preference. Here are the results:

rank												
1	A	A	A	A	A	C	C	C	B	B	B	C
2	B	B	B	B	C	B	B	B	C	C	C	A
3	C	C	C	C	B	A	A	A	A	A	A	B

- 1 Which choice is the least popular?
- 2 Where should they go in order to please the most people?
  - 1 Is there a convincing argument that they should go to option A?
  - 2 Is there a convincing argument that they should go to option B?
  - 3 Is there a convincing argument that they should go to option C?
- 3 They learn that the Chinese restaurant is closed. Does this change your answers to questions (a) and (b) above?

## Theorem (Arrow's Impossibility Theorem)

*When voters have three or more distinct alternatives (options), no ranked voting electoral system can convert the ranked preferences of individuals into a community-wide ranking while also meeting a specified set of criteria: non-dictatorship, Pareto efficiency, and independence of irrelevant alternatives.*



The end.

Thank you.