MATH 322/323 Module 1 Cartesian Tensors Mar 5 – May 5 2014
Assignment 3 and Tutorial 4

Timetable-whole term--modified

Week	1	2	3	4	5	6	7
Start	Mar 3	Mar 10	Mar 17 Assignment 1 due	Mar 24	March 31	Apr 7 Assignment 3 due	April 14
Mon 14:10-15:00	Intro lecture	L3	L5	L7	L9	Spare	L11
Tues 14:10-15:00	L1 Assignment 1 set	L4 Assignment 2 set	L6	L8	L10 Assignment 4 set	spare	T6
Weds 14:10-15:00	L2	Spare	Spare	Spare Assignment 2 due	Spare	Spare	Spare
Tutorial Fri 14:10-15:00	T1	T2	T3 Assignment 3 set	T4	T5	Spare	Assignment 4 due Thursday 17 April

Assignments and tutorial exercises

All assignments due 5pm on day of week shown.

Essay due 5pm Monday 5 May

Assessment Summary

Assignment 1	20%	Index notation; Rotational transformations; Euler vector
Assignment 2	20%	Prove Kronecker is a tensor; lead rubber bearing, stress force across a plane
Assignment 3	20%	Strain gauges – principal axes, simple shear
Assignment 4	20%	Hooke's Law, tensor calculus
Essay	20%	**due 5 May.**

MATH/GPHS 322/323 Tensors Module

Assignment 3 ***due 7 April*.**

(1) a, b, c are three strain gauges 60° apart all in the same plane (c.f. Q1 in the tutorial exercises, where a, b, c are 45° apart).

If ε_a , ε_b , ε_c , are the strains measured in the directions a, b, c respectively, find the 2-D strain tensor, the Principal Strains and the directions of the Principal Axes.

Hint. Solve the quadratic equation for the eigenvalues using tensor components of strain before you substitute for ε_a , ε_b , ε_c . Do not spend a lot of time trying to simplify the resulting expression.

(2) A continuum deforms as follows: the displacement Δu_i of any point P relative to the origin is of the form:

$$\Delta u_i = (u_1, 0, 0)^T ,$$

where $u_1 = k\, a_2$, where k is a constant << 1, and a_1, a_2, a_3 are Lagrangian coordinates (this deformation is called *Simple Shear*).

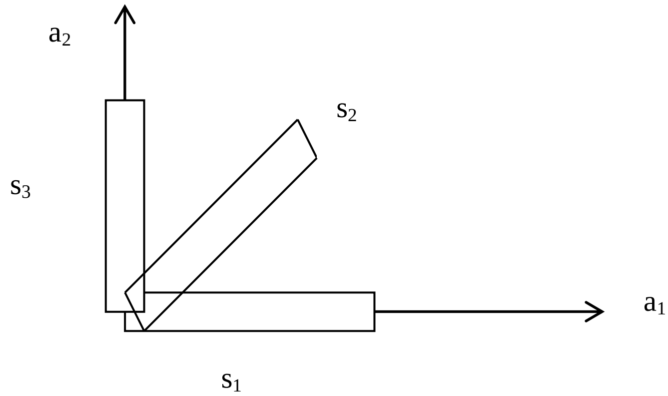
Find the Strain and Rotation tensors E and W, the equivalent rotation vector $\underline{\boldsymbol{\omega}}$, and the Principal Strains and Principal Axes of E. What is the dilatation?

(3) Find the Principal Axes, Principal Strains and dilatation for a continuum where the strain tensor is:

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -2 \end{bmatrix} \times 10^{-4}$$

Tutorial Four 28 March (to help with Assignment 3)

(1) We have three strain gauges (ie instruments which measure the change in a length of wire, or the distance between two points using a laser, etc) deployed 45° apart in a plane, as shown.



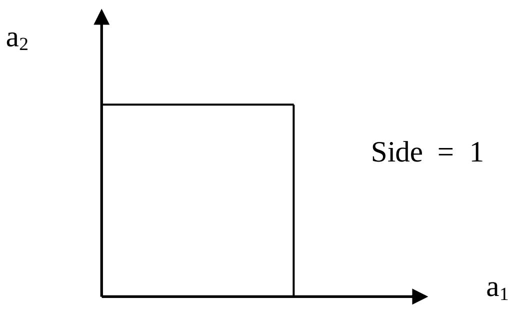
The material the strain gauges are mounted on suffers a strain E and each registers a (linear) strain = ε_1, ε_2, ε_3 respectively. That is, there is a change of :

ε_1 per unit length in the direction of s_1, which is a_1;
ε_2 per unit length in the direction of s_2, which is at 45 ° to a_1;
ε_3 per unit length in the direction of s_3, which is a_3;

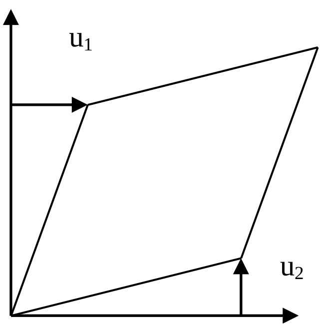
Without loss of generality, we can take each strain gauge to have unit length. For each of s_1, s_2, s_3 take their end-points to be on a unit circle.

Calculate the strain tensor E using strain gauge measurements.

(2) Consider a unit square: which deforms to:



Side = 1



There is no deformation, or change, in the a_3 direction, so we have *plane strain*.
The rules that determine u_1 and u_2 are that $u_1 \propto a_2$ and $u_2 \propto a_1$ with *different* constants of proportionality k; so:

$u_1 = k_1 a_2$ and $u_2 = k_2 a_1$

Find the Strain and Rotation tensors, the equivalent rotation vector, the Principal Strains, the Principal Axes and the dilatation.

NB example with $k_1 = k_2$ is in lectures.