MATH 322/323 Module 1 Cartesian Tensors Mar 5 – May 5 2014
Assignment 4 and Tutorial 5 Revised Schedule:

Timetable

Week	1	2	3	4	5	6	7
Start	Mar 3	Mar 10	Mar 17 Assignment 1 due	Mar 24 Assignment 2 due	March 31	Apr 7 Assignment 3 due	April 14
Mon 14:10-15:00	Intro lecture	L3	L5	L7	L9	Spare	T5
Tues 14:10-15:00	L1 Assignment 1 set	L4 Assignment 2 set	L6 Assignment 3 set	L8 Assignment 4 set	L10	spare	T6
Weds 14:10-15:00	L2	Spare	Spare	Spare	L11	Spare	Spare
Tutorial Fri 14:10-15:00	T1	T2	T3	T4	L12	Spare	Assignment 4 due Thursday 17 April

Assignments and tutorial exercises

All assignments due 5pm on day of week shown.

Essay due 5pm Monday 5 May

Assessment Summary

Assignment 1	20%	Index notation; Rotational transformations; Euler vector
Assignment 2	20%	Prove Kronecker is a tensor; lead rubber bearing, stress force across a plane
Assignment 3	20%	Strain gauges – principal axes, simple shear
Assignment 4	20%	Hooke's Law, tensor calculus
Essay	20%	**due 5 May.**

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Assignment 4 ***due Thursday 17 April.***

(1) Assuming the form of Hooke's Law for an isotropic material:

$$S_{ij} = 2\mu E_{ij} + \lambda E_{kk} \delta_{ij}$$

(i) Show that the Bulk Modulus K defined to be $1/3\ S_{kk} / E_{kk}$ is given by $K = \lambda + 2/3\ \mu$

Young's modulus Y is measured as the ratio of a uniaxial tension S_{11} to the strain E_{11} it produces in a body ('uniaxial' means that S_{22} and $S_{33} = 0$).

(ii) Write down the equations for S_{11}, S_{22} and S_{33} from Hooke's Law.

(iii) Under uniaxial tension, the body will contract in the x_2 and x_3 directions i.e. $E_{22}, E_{33} \neq 0$.

Poisson's Ratio ν is defined by: $\nu = - E_{22} / E_{11}$

Assuming that the body has axial symmetry, show that

$$\nu = \lambda / 2(\lambda + \mu)$$

(iv) Show that: $Y = \mu (3\lambda + 2\mu)/(\lambda + \mu)$

NB by making appropriate measurements of K, Y and ν, we can infer the Lame Constants for a material.

(2) If E_{ij} and S_{ij} are the strain and stress tensors in a continuum, the *strain potential energy* W per unit volume of the material is defined to be the work done in straining a unit volume of material to strain E_{ij}:

$$W = \int_0^{E_{ij}} S_{kl}\, dE_{kl} \quad \text{(summation convention).}$$

(i) Show that for an elastic material

$$W = \lambda/2\ E_{kk}^{\ 2} + \mu E_{kl} E_{kl}$$

3. (i) Show that $u_1 = A \sin(\omega t \pm \nu x_1)$; $u_2 = 0$; $u_3 = 0$

where t is time and A, ω and ν are constants, is a solution of Navier's equation without body forces viz.

$$\rho\, \partial^2 u_i / \partial t^2 = \mu\, \partial^2 u_i / \partial x_j \partial x_j + (\mu + \lambda)\, \partial^2 u_k / \partial x_k \partial x_i$$

provided $c = \omega/ \nu$ satisfies

$$c^2 = (K + 4/3\ \mu)/\rho \qquad \text{where K is the Bulk Modulus.}$$

(ii) *Briefly* describe the way in which the continuum is deforming in this motion –

- In space (fix time)
- In time (fix position x_1).

(iii) If $(\omega t \pm \nu x_1)$ is dimensionless, what are the dimensions (units) for ω, ν and c? What is the physical meaning of c?

Tutorial Five 14 April (and 15 April if required)

1. A specimen of isotropic material is subjected to compression S_{11}, but is constrained so that it cannot expand in the x_2 and x_3 directions. Show that the apparent modulus of elasticity is:

$Y(1 - \nu)/(1 + \nu)(1 - 2\nu)$

Hence show that we can re-write Hooke's law for isotropic elastic solids as:

$$E_{ij} = (1 + \nu)/Y \ S_{ij} - \nu/Y \ S_{kk} \delta_{ij}$$

2. If E_{kl} is defined by $E_{12} = E_{21} = -2$, $E_{kl} = 0$ otherwise, evaluate

$$I = \int_0^{E_{kl}} U_{ij} \, dU_{ij}$$

3. Show that:

$u_2 = A \sin(\omega t \pm \nu x_1)$; $u_1 = 0$; $u_3 = 0$

is a solution of Navier's equation without body forces -

$$\rho \, \partial^2 u_i / \partial t^2 = \mu \, \partial^2 u_i / \partial x_j \partial x_j + (\mu + \lambda) \, \partial^2 u_k / \partial x_k \partial x_i$$

provided $c = \omega/\nu$ satisfies

$$c^2 = \mu/\rho$$