MATH/GPHS 322, 323 2014

Module on Fluid Flow in Earth Systems, Assignment 2. Due Friday 19 Sept. 5 PM in my office (Murphy 1114 11th floor) or assignment box next to SGEES admin office, 3rd floor, Cotton Building

Reading: Turcotte & Schubert, Course notes. Please read the sections outlined below and do the nine (9) problems below.

Marking: You do not need to necessarily get the correct answer to get some credit. However, you also will not necessarily get credit for getting the correct answer if you don't show how you did it. To get full credit (10 marks for each problem), you must do the following:

1) Draw a figure explaining the problem, with the coordinate system and any symbols explained. (2 marks)
2) Any equation you use must be either referenced, e.g., with the equation number from Turcotte & Schubert, stating which edition (the edition can be stated once at the start of the assignment) or else derived from a preceding equation with any non-obvious steps explained. (1 mark)
3) Highlight the answer at the end in some fashion, e.g., underline or box or label ANSWER. (1 mark)

The last 6 marks are for the proper working of the problem.

Read sections 6-7 Conservation of Fluid in Two Dimensions, 6-8 Elemental Force Balance in Two Dimensions and 6-9 The Stream Function and do problem 6-10.

1. 6-10: Determine the stream function ψ for the general one-dimensional channel flow discussed in section 6-2. Find ψ for the asthenospheric counterflow model in section 6-3. Discuss the physical meaning of ψ in all cases.

Read section 6-10 Postglacial Rebound and do problems 6-11 and 6-12.

2. 6-11: Show that the constant of integration A in the postglacial rebound solution given in this section is given by $A = -\left(\frac{\lambda}{2\pi}\right)^2 \frac{\rho g w_{m0}}{2\mu} e^{-t/\tau_r}$.

3. 6-12. Modified as described here. The ice sheet over Hudson Bay, Canada, had an estimated thickness of 2 km. At the present time there is a negative free-air gravity anomaly in this region of 0.3 mm s^{-2}.

a. Assuming that the ice (density of 1000 kg m^{-3}) was in isostatic equilibirium and displaced mantle rock with a density of 3300 kg m^{-3}, determine the depression of the land surface w_{m0}. Hint: The free air gravity anomaly is the difference in the gravity measured at a location from the gravity expected for an average earth model at the given location (latitude, longitude and height).
b. Assuming that the negative free-air gravity anomaly is due to incomplete rebound, determine *w* at the present time. Hint: The correction to gravity due to the attraction of a body due to its topography is given by the *Bouguer gravity formula*, $\Delta g = 2\pi\rho G h$ where *h* is the height of the topography, *G* is the universal gravitational constant, G=6.67x10-11$m^3kg^{-1}s^{-2}$ and ρ is the density of the body.
c. Applying the periodic analysis given in section 6-10, determine the mantle viscosity. Assume that the ice sheet melted 10,000 years ago and that the appropriate wavelength for the Hudson Bay ice sheet was 5000 km.

d. (replacement) Compare your results and the analysis in the book of the uplift at Hudson's Bay with the paper by Sella et al. in Geophysical Research Letters, 2007 in your course notes. How close are the assumptions made in the book to the more detailed analysis available today (e.g., rates of uplift at different places)? How does the viscosity you determine with such a simplified model compare with the viscosity structure proposed in this paper?

Section 6-11 Angle of Subduction

4. Problem 6-14: Derive expressions for the lifting torques on the top and bottom of a slab descending into the mantle with speed *U* at a dip angle of 60°.

Section 6-12 Diapirism

5. Problems 6-15: A layer of salt at a depth of 3 km with a density of 2150 kg m^{-3} lies beneath sediments with a density of 2600 kg m^{-3}. The salt layer is known to have doubled the amplitude of its instability in 100,000 years. Estimate the equivalent viscosity of the system.

6. Problem 6-17: Discuss how you would modify the analysis of this section to account for a viscosity difference between the two fluid layers.

Section 6-13 Folding. Read it if you like, but it's not necessary for the later sections, and not included in the handout.

Section 6-14 Stokes Flow.

7. Problems 6-23: Consider a spherical bubble of a low-viscosity fluid with density ρb rising or falling through a much more viscous fluid with density ρf and viscosity μf because of a buoyancy force. For this problem the appropriate boundary conditions at the surface of the sphere, *r*=*a*, are u_r=0 and $\tau_{r\theta}$=0. Using Equations 6-10, (6-211) and (6-220) show that

$$u_r = U(-1+\frac{a}{r})\cos\theta$$

$$u_\theta = U(-1+\frac{a}{2r})\sin\theta$$

By integrating Equation (6-196), show that on *r*=*a*,

$$p = \frac{\mu_f U}{a}\cos\theta$$

The drag force is obtained by carrying out the integral

$$D = 2\pi a^2 \int_0^\pi (p - 2\mu_f \frac{\partial u_r}{\partial r})_{r=a} \cos\theta \sin\theta d\theta$$

Show that $D = 4\pi\rho_f aU$

And demonstrate that the terminal velocity of the bubble in the fluid is

$$U = \frac{a^2 g(\rho_f - \rho_b)}{3\mu_f}$$

8. Former VUW Professor Christoffel calculated the viscosity of the mantle assuming that the deep earthquakes under Mt. Taranaki that occur at a depth

of 600 km are caused by a piece of slab that broke off and fell through the mantle following Stoke's law (Christoffel, 1973: Nature Physical Science, Vol. 243, No. 125, pp. 51-52, May 21, 1973—see course notes). He assumed that the broken slab acted like a falling sphere with radius 30 km. He calculated the velocity *v* based on the assumption that the fragment fell 380 km in 3.8 My. Assuming further that the density difference between the slab and the surrounding mantle is 50 kg m^{-3}, determine the viscosity of the mantle (Be careful about units!!). How does it compare to the accepted values in Turcotte and Schubert?

9. Calculate the rising or sinking velocity of a 'drop' of iron of radius 50 km and average density excess 5 Mg/m^3 in a hot mantle with an average viscosity of 10^{20} Pa s. How many years would it take the drop to reach the core?