Fluids module 2014

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- Motto: There's no such thing as a stupid question

Notes for Assignment 1
Gphs/Maths 323 2014
Fluid Flow in Earth Systems

1) Reading/Tutorial/Assignment course
2) Important reading points highlighted
3) But need to read yourself.

First meeting

- Meetings are scheduled as12-1 Mon Tues Wed Friday. Probably only need 3 meetings.
- Clashes Mondays and Wednesdays—what to do?
- Assignments all received/email correct?
- Introduction / text book / Maths vs. Gphs
- Program—3 weeks analytical; 3 computer

- Text for first 3 weeks: Turcotte & Schubert—either edition-important parts are copied and for sale at bookstore
- Tutorials will
  - Cover basic concepts that may not be well explained in the book
  - Specifically answer questions about assignment problems

- Pre-course quiz—please add preferred email to name

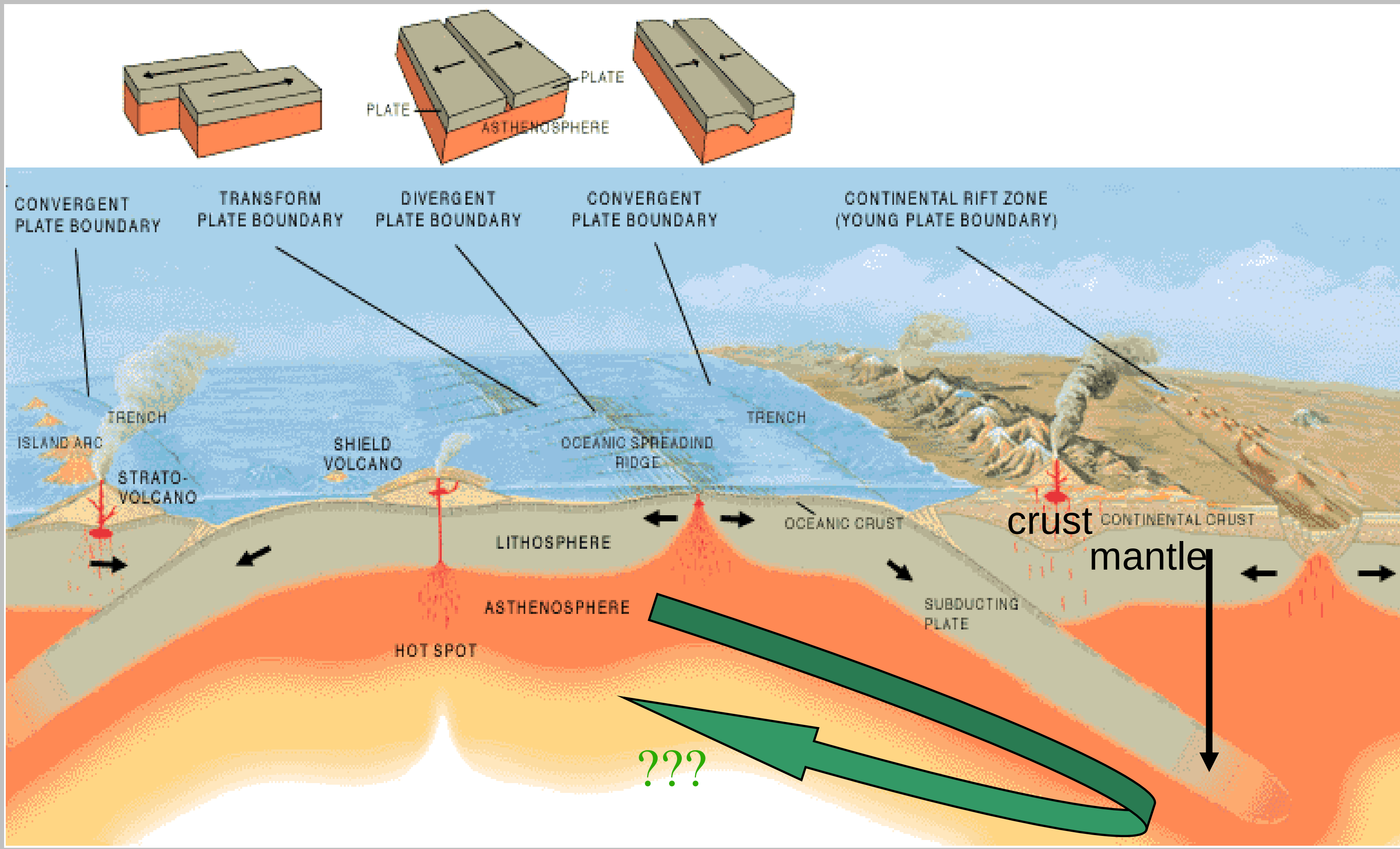


PLATE
PLATE
ASTHENOSPHERE
CONVERGENT PLATE BOUNDARY
TRANSFORM PLATE BOUNDARY
DIVERGENT PLATE BOUNDARY
CONVERGENT PLATE BOUNDARY
CONTINENTAL RIFT ZONE (YOUNG PLATE BOUNDARY)
TRENCH
ISLAND ARC
STRATO-VOLCANO
SHIELD VOLCANO
OCEANIC SPREADIND RIDGE
TRENCH
OCEANIC CRUST
crust
CONTINENTAL CRUST
mantle
LITHOSPHERE
ASTHENOSPHERE
HOT SPOT
SUBDUCTING PLATE
???

What is a fluid?

- Material in which strain rate depends on stress
- Newtonian Fluid: Stress $\propto$ Strain rate
- (Non-newtonian: Strain rate $\propto$ (stress)^n where n >1)
- 3-D: 1-D:

$$\overleftrightarrow{\tau} = \mu \dot{\overleftrightarrow{\varepsilon}}$$

EQ 6-1 $$\tau = \mu\dot{\varepsilon} = \mu \frac{du}{dy}$$

τ=shear stress: dot denotes time derivative

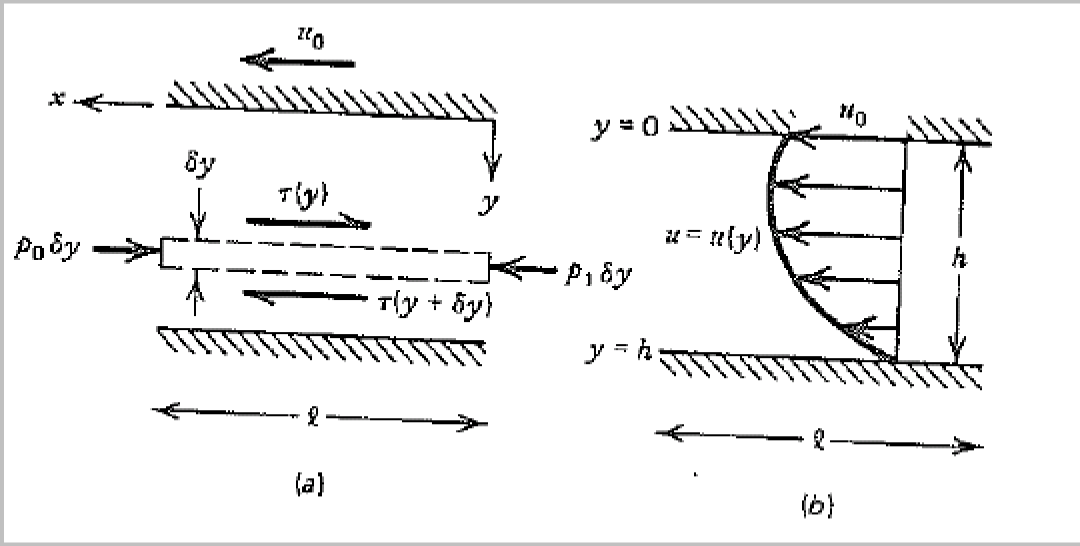
ε=strain; μ=viscosity; u=velocity; x,y=position

1-Dimensional equations

- “Real” equations are 3-D
- Book often uses 1-D approximation
- This means that properties vary only in one dimension
- Other two dimensions are homogeneous
- Other dimension may be “implied” in equation.

Example, Fig. 6-1: 1-D channel flow

- τ=shear stress
- Units Force/area=Pa
- Book: Force on upper boundary is
- $-\tau(y)\ell$
- Missing dimension-into page: (assume unity)



p=pressure
u=velocity
x,y=position
ℓ =length

Pressure, velocity are applied to fluid and motion is to be determined

Really
$F=-\tau(y)\ell z$
Where F=force
z=length of 1 into page

Why is the last equality true?

-

- 3-D: 1-D:

$$\overleftrightarrow{\tau} = \mu \dot{\overleftrightarrow{\varepsilon}}$$

EQ 6-1 $$\tau = \mu \dot{\varepsilon} = \mu \frac{du}{dy}$$

τ=shear stress: dot denotes time derivative
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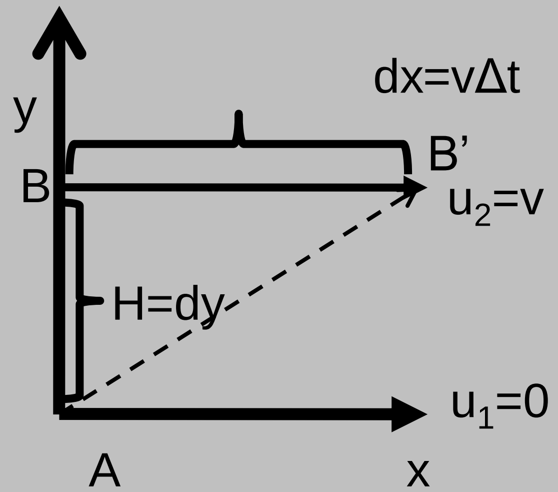
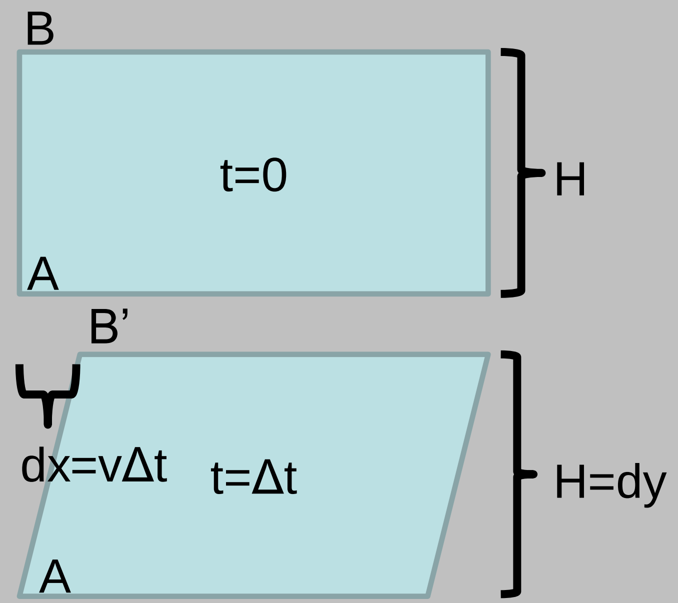
1-D

- Deformation = strain=dx/dy. Change in Shape/time = deformation rate=d/dt(dx/dy):





- In time Δt, point A constant: Point B at height y=H moved to B' at v Δt. Shape change = $dx/dy = v\,\Delta t/H$
- Shape change/time = $v\,\Delta t/H\Delta t = v/H$ = velocity gradient =change in velocity/space

- Strain rate = Shape change/time = v Δt/HΔt = v/H = velocity gradient =change in velocity/space

EQ 6-1 $$\tau = \mu \dot{\varepsilon} = \mu \frac{du}{dy}$$

3-D:

$$\overleftrightarrow{\tau} = \mu \dot{\overleftrightarrow{\varepsilon}}$$

Units?

$\overleftrightarrow{\tau} = \textit{stress}$ tensor

$\dot{\overleftrightarrow{\varepsilon}} =$ strain rate tensor

$\mu =$ viscosity

3-D:

$$\overleftrightarrow{\tau} = \mu \dot{\overleftrightarrow{\varepsilon}}$$

$\vec{w} = \text{displacement}$

$\vec{u} = \text{velocity}$

$$\tau_{ij} = \mu \dot{\varepsilon}_{ij}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial w_i}{\partial x_j} + \frac{\partial w_j}{\partial x_i}\right)$$

$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial \dot{w}_i}{\partial x_j} + \frac{\partial \dot{w}_j}{\partial x_i}\right) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

(3-D equations become most important in cylindrical and spherical geometries/coordinates)

Viscosity

- Dynamic viscosity: μ units: Pa-s
- Sometimes consider Kinematic viscosity ν
- Greek letter nu, not v
- Units Pa-s/(kg/m^3)
- =m^{2}s^{-1}

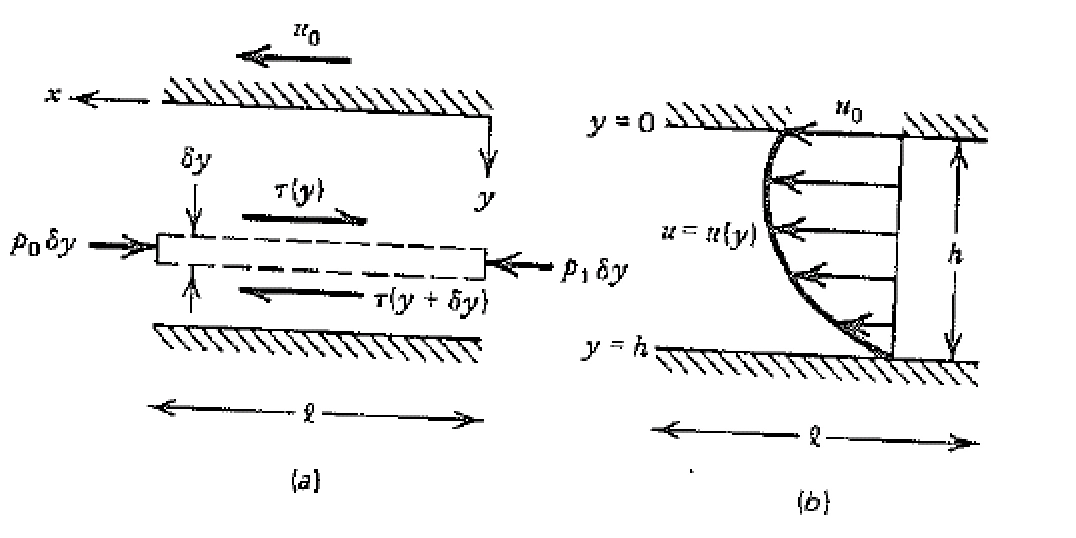
$$\nu = \frac{\mu}{\rho}$$

- (How rapidly does momentum diffuse?)

Dimensionless Numbers

- Common in Fluid Mechanics
- Allow general problems to be studied which can be applied to many scales
- But can be confusing.
- Prandtl Number: $\mathrm{Pr} = \frac{\nu}{\kappa} = \frac{\text{kinematic viscosity}}{\text{thermal diffusivity}}$
- Small Pr➔ heat diffuses more rapidly than momentum

1-D Channel flow solutions:



p=pressure (note: not same as ρ=density)
Pressure gradient in x-direction=dp/dx
u=velocity in x direction

EQ 6-1

$$\tau = \mu \frac{du}{dy}$$

Start with this (equation of motion-from balance of forces)

$$\frac{d\tau}{dy} = \frac{dp}{dx} \quad (6-8)$$

Substitute Eq 6-1
To get this

$$\mu \frac{d^2u}{dy^2} = \frac{dp}{dx}$$

Integrate to get this

$$u = \frac{1}{2\mu}\frac{dp}{dx} y^2 + C_1 y + C_2$$

Is this the answer?

Important concepts for solving differential equations

- Solution to the equation depends on the **boundary conditions**
- So far, have:
- 1. No-slip condition: Viscous fluid in contact with a solid boundary must have same velocity at the boundary
- 2. Free Surface: $\tau = 0$ at surface (No shear stress at surface)

Example-last slide shown

$$\frac{d\tau}{dy} = \frac{dp}{dx} (6-8)$$

Starting Equations (pressure gradient causes gradient in shear stress); Stress is viscosity times strain rate or velocity gradient

$$\tau = \mu \frac{du}{dy} (6-1)$$

Differentiate wrt y and substitute ➔

$$\frac{d\tau}{dy} = \mu \frac{d^2u}{dy^2} = \frac{dp}{dx}$$

Integrate once➔

$$\frac{du}{dy} = \frac{1}{\mu}\frac{dp}{dx} y + C_1$$

If there is a free surface: Use this eqn to evaluate C_1

$$(\frac{du}{dy} = \frac{\tau}{\mu} = \frac{1}{\mu}\frac{dp}{dx} y + C_1 \; (\tau = 0 \text{ at some y position})$$

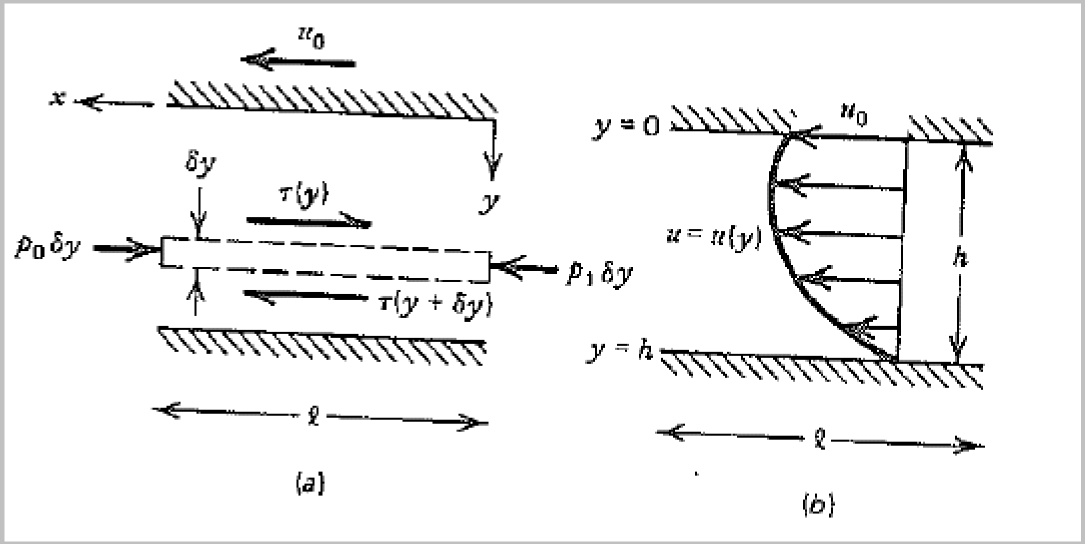
$$u = \frac{1}{2\mu}\frac{dp}{dx} y^2 + C_1 y + C_2$$

Integrate twice: Use no-slip condition here ($u=u_0$ at some y position)

Example

- Also, get intermediate Solution: $\Delta P=\rho gH$
- Where H=Hydraulic head and ΔP=pressure difference:
- Difference in pressure depends only on height difference
- True for tubes—e.g., siphons, and also for reservoirs and water tanks.

General solution for 1-D flow



- Equation 6-12: $$u = \frac{1}{2\mu}\frac{dp}{dx}(y^2 - hy) - \frac{u_0 y}{h} + u_0$$
- If dp/dx=0; Couette Flow
- If u=0, dp/dx≠0 no special name-just stationary boundary condition.

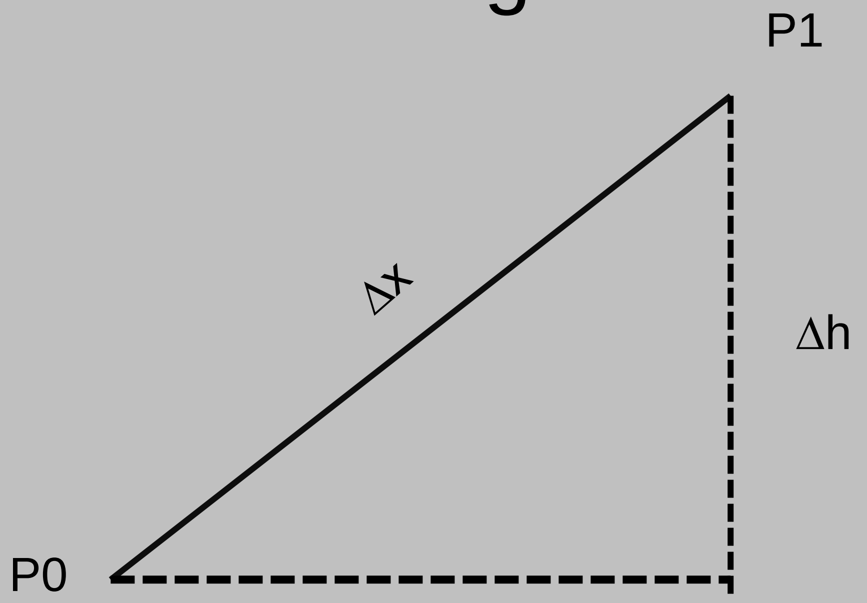
Problem hints

- Draw pictures!
- Consider boundary conditions!

- Remember 1st year calculus—how do you get the average of a function?
- How do you get the maximum and minimum of a function?

Fluid on an inclined plane

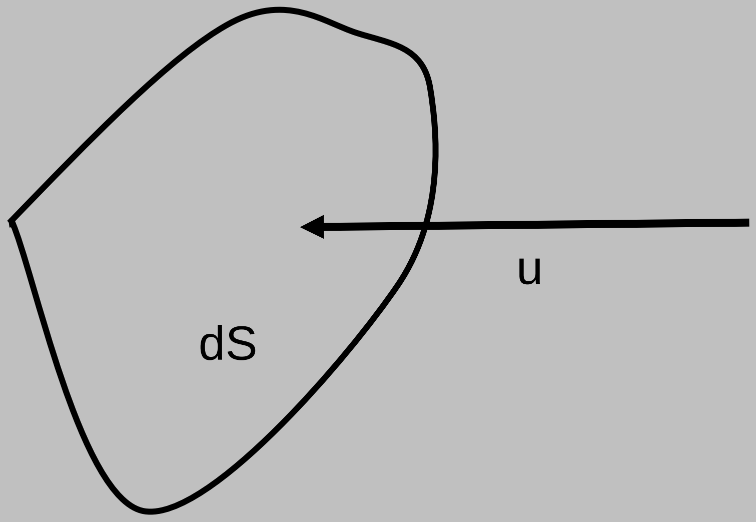
- Difference in pressure depends only on difference in height



Volumetric flow rate

- Q = volumetric flow rate=total volume of fluid passing a cross-section per unit time.
- Examples: River, pipe

$$Q = \int_{surface} udS$$

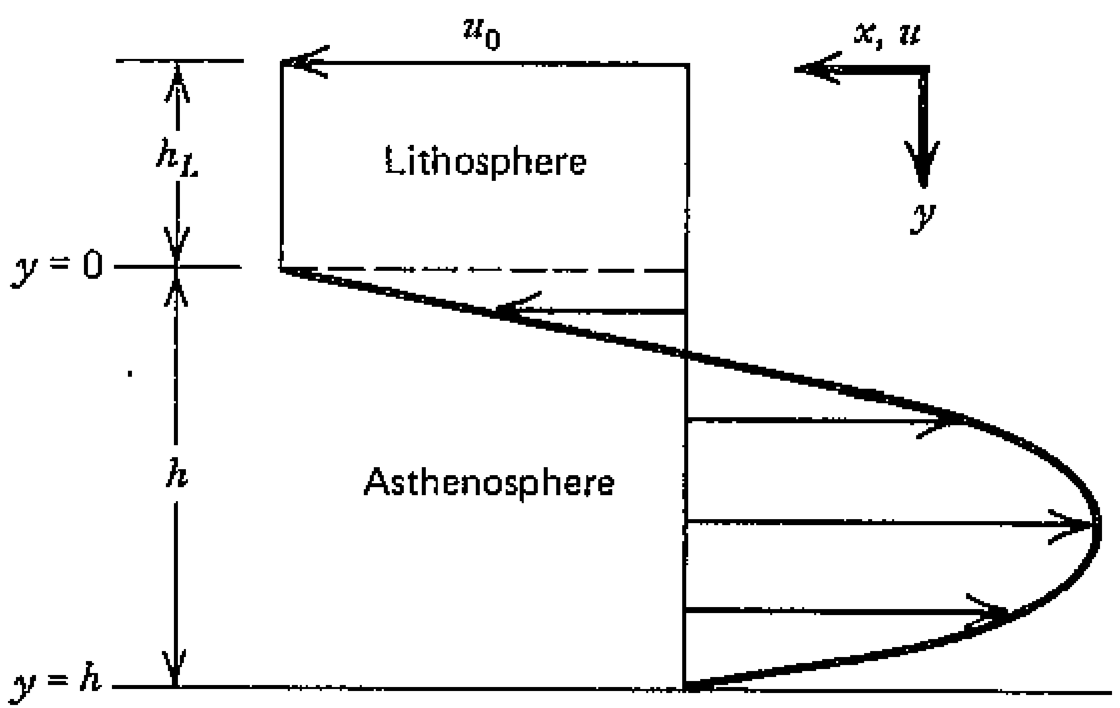


- (u is component perp.

to surface)

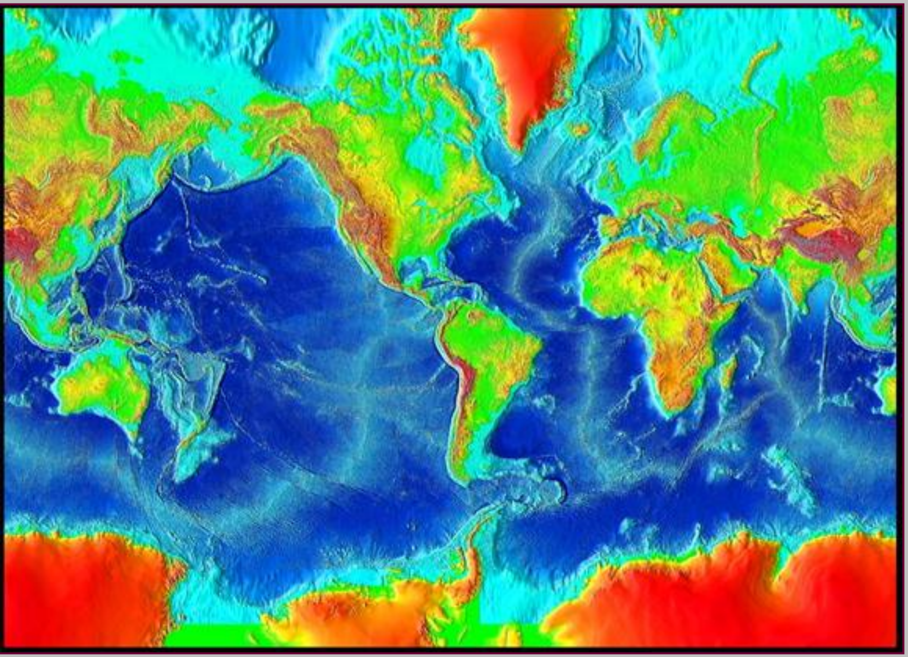
Asthenospheric counterflow

- People originally thought it might exist



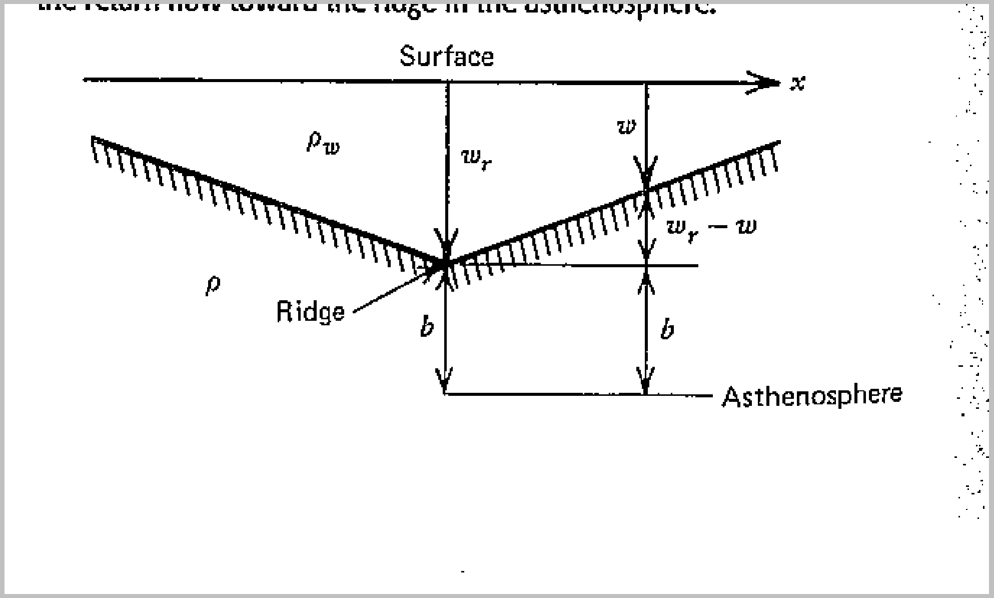
6–4 Velocity profile associated with the asthenospheric counterflow model.

But—model prediction of sea floor topography is opposite to what is observed

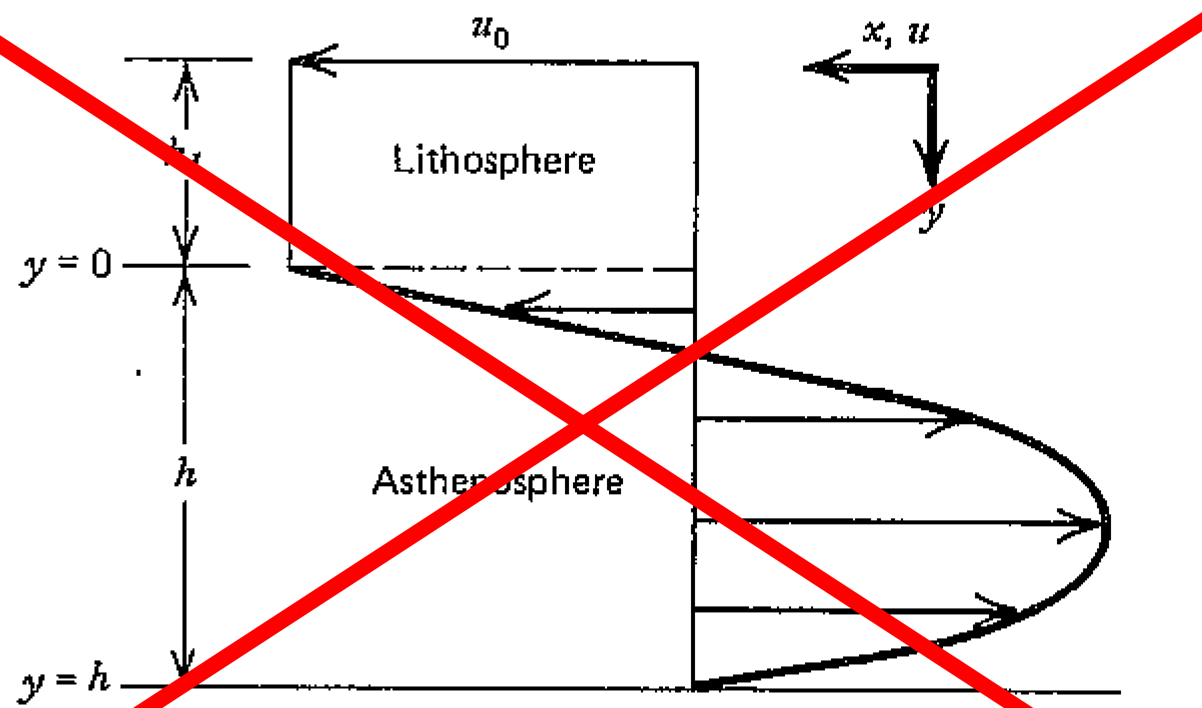


Darker blue = deeper ocean
Lighter blue=shallower

Model Prediction



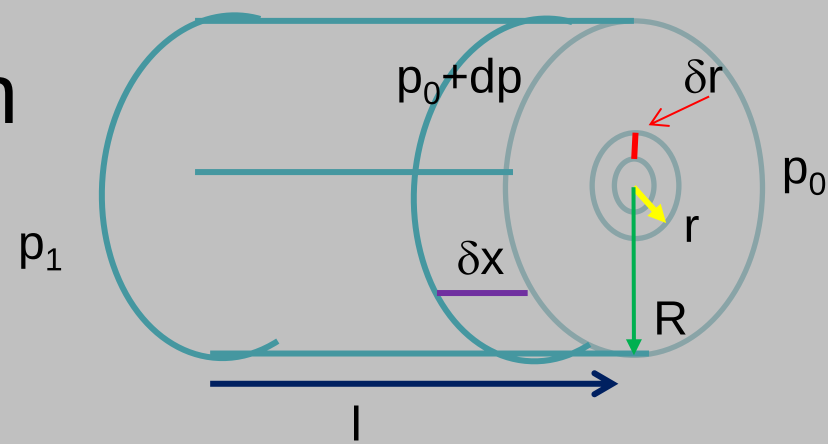
So Theory is wrong



6–4 Velocity profile associated with the asthenospheric counterflow model.

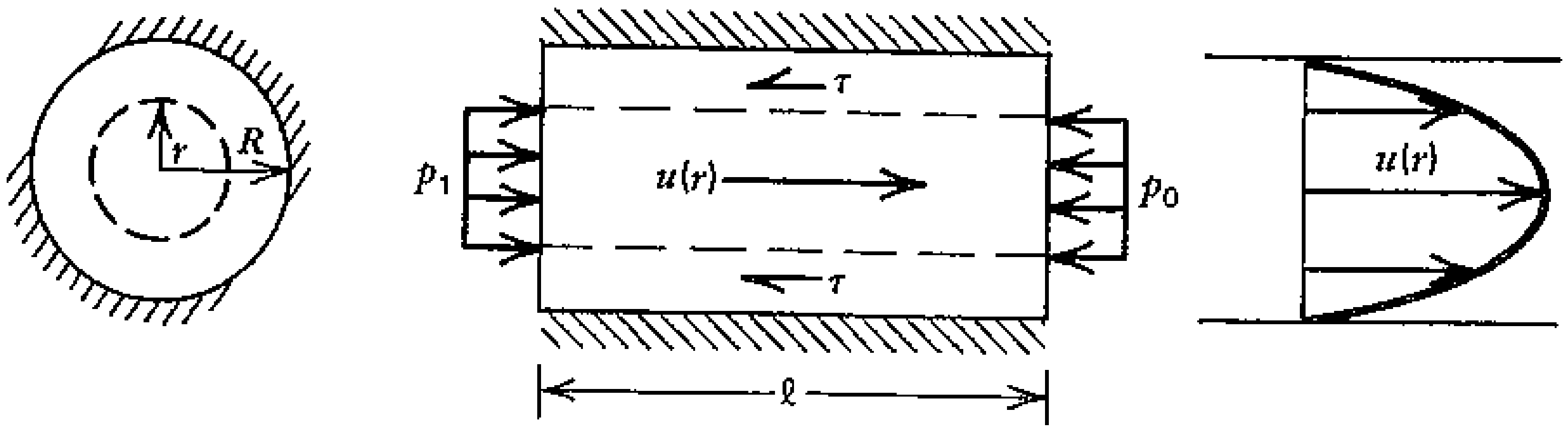
6-4 Pipe Flow

3-D view



- Poiseuille flow through a circular pipe
- Fig. 6-6

r pipe.



Equations :

$$\tau = \frac{r}{2}\frac{dp}{dx} = \mu\frac{du}{dr}$$

Integrate to get:

$$u = -\frac{1}{4\mu}\frac{dp}{dx}(R^2 - r^2)$$

Reynold's Number Re: dimensionless

- D=dimension of problem (e.g., pipe diameter)
- μ=dynamic viscosity
- ν=kinematic viscosity

$\bar{u} = \text{avg speed}$

$$\mathrm{Re} = \frac{\rho \bar{u} D}{\mu} = \frac{\bar{u} D}{\nu}$$

Re>2200➔turbulent flow

- Re<2200➔laminar flow
- Re<1➔ Stokes flow = reversible—movie

- http://web.mit.edu/fluids/www/Shapiro/ncfmf.html.