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MATH 321/322/323 Applied Mathematics T1 and T2 2013

# Module on Special Relativity: Assignment 1

This assignment covers chapter 1 ("Spacetime: Overview").
Remember:

$$c = \text{(speed of light)} = 3 \times 10^8 \text{ metres/sec} = 3 \times 10^5 \text{ km/sec}$$

1. [Invariant interval]

   As seen by one observer, two firecrackers are set off at the same place but separated by 4 seconds in time.

   As observed by a second observer, moving with respect to the first, the two firecrackers detonate at places separated by 3 light-seconds in space.

   As observed by this second observer, how much time is there between the two detonations?

   (Hint: Don't use a calculator. The answer is an integer, and you should be able to do this problem without electronic aids of any kind.)

   How fast is this second observer moving with respect to the first?

   (Hint: A rational number [fraction] is good enough.)

2. [Invariant interval]

   A pulse of laser light leaves Earth and arrives at Sun about 8 minutes later.

   - What is the invariant interval between the emission event and the collision event?
   - Without looking anything up, what's the approximate size of the astronomical unit (AU) in metres?

   (Hint: It's *really* easy.)

3. [Speed of light limitations in computer design]

   As of early 2013, it's reasonably easy to buy home computers powered by CPUs running with a clock speed of 3 GHz. So let's assume you have a home computer powered by a CPU running at a speed of 3 GHz.

   - How long does each "clock cycle" take in seconds?
   - How far can light travel in one "clock cycle"?
   - How far can an electrical impulse travel in one "clock cycle"?
   - Assume that in each clock cycle, to complete whatever task it is doing, the chip has to designed in such a way that it is possible to get an electrical impulse from any part of the chip to any other part of the chip and back again. Given this design constraint, what is the maximum possible diameter for a 3 GHz computer chip?
   - Additionally, the CPU needs some data to work on — if that data has to be read out of memory, processed, and put back into memory before the next clock cycles starts, what's the maximum distance between any part of the chip and any part of the memory?
   - How close are the engineers to reaching this speed of light limit?
   - What would happen if we re did the calculation for a 1 THz clock speed (1 Tera-Hertz)?

   Comment: One way of trying to evade the speed of light limit is with multiple CPUs talking to each other asynchronously. This is the central idea behind parallel computing.

4. [Time stretching for muons]

   At heights of 10 to 60 kilometers above the surface of the Earth, cosmic rays continually strike nuclei of oxygen and nitrogen atoms in the upper reaches of the atmosphere and produce muons (elementary particles of a mass about 207 times that of the electron).

   Some of the muons move vertically downward with a speed nearly that of light. Follow one of the muons on its way down. In a given sample of muons, half will decay to other elementary particles in 1.5 microseconds ($1.5 \times 10^{-6}$ seconds), as measured in a reference frame in which they are at rest. Half of the remainder decay in the next 1.5 microseconds, and so on. Analyze the results of this decay as observed in two different frames.

Idealize the rather complicated details of the actual experiment to the following roughly equivalent situation: All muons are produced at the same height (45 kilometers); all muons have the same speed; all travel straight down; none are lost to collisions with air molecules on the way down.

- Approximately how long a time will it take these muons to reach the surface of the Earth, as measured in the Earth frame?
- If the decay time were the same for Earth observers as for an observer travelling with the muons (that is, ignoring time dilation), approximately how many half lives would have passed by the time the muons reach the surface?
- Therefore what fraction of those created at a height of 45 kilometers would remain once they reach the surface if the Earth? (An answer of the form 1/2 raised to some suitable power is good enough.)
- An experimenter determines that the actual fraction of muons which reaches sea level is 1/16. In the muon's rest frame, how many half lives have elapsed between creation of a given muon and its arrival at the surface of the Earth?
- In the muon's rest frame what is the space separation between the birth of a survivor muon and the place where it arrives on the surface of the Earth?
- Find the value for the spacetime interval between the birth event and the arrival event.
- By what factor is time dilated for these moving muons?

5. [Time stretching for pions]

   Laboratory experiments on particle decay are for technical reasons much more conveniently done using $\pi^+$ mesons (positively charged pions; pi-plus mesons) rather than $\mu$ mesons (mu-mesons, muons).

| Particle | Half-life (rest frame) | $c\times$(half-life) |
|---|---|---|
| $\mu$-meson<br>muon<br>(207 electron masses) | $1.5 \times 10^{-6}$ seconds | 450 metres |
| $\pi$-meson<br>pion<br>(273 electron masses) | $18 \times 10^{-9}$ seconds | 5.4 metres |

For more than you ever wanted to know about elementary particles, see http://pdg.lbl.gov

In a given sample of $\pi^+$-mesons, half will decay to other elementary particles in 18 nanoseconds ($18 \times 10^{-9}$ seconds), as measured in the reference frame in which the $\pi^+$-mesons are at rest. Half of the remainder will decay in the next 18 nanoseconds, and so on.

- In a particle accelerator $\pi^+$-mesons are produced when a proton beam strikes an aluminium target inside the accelerator. Mesons leave this target with nearly the speed of light. If there were no time stretching, and if no mesons were removed from the resulting beam by collisions, what would be the greatest distance from the target at which half of the mesons would remain undecayed?
- The $\pi^+$-mesons of interest in a particular experiment have a speed of 0.9987 tines the speed of light. By what factor is the predicted distance from the target for half-decay increased due to the time dilation effect? That is by what factor does the time dilation effect allow one to increase the distance between the detecting equipment and the target?

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