School Of Mathematics, Statistics, and Operations research
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MATH 321/322/323 Applied Mathematics T1 and T2 2013

Module on Quantum Mechanics: Assignment 4

- This fourth and final assignment will deal with the S-matrix.
- Read chapter 6 of the notes — the chapter on the S-matrix.
- Let me know of any typos.

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1. For a general potential, evaluate the determinant of the S matrix — simplify it as much as possible.

   It will be useful to write it in terms of the phase ϕ_0 of the transmission amplitude t.

   Notation: Remember that for any arbitrary complex number we have $z = x + iy = re^{i\phi}$.

   The modulus is $r = \sqrt{x^2 + y^2}$ and the phase is $\phi = \tan^{-1}(y/x)$.

2. Show that for any arbitrary potential the S-matrix is always *unitary*.

   How is this related to the conservation of flux?

   Remember the adjoint (Hermitian conjugate) is *defined* by

   $$S^\dagger = (S^*)^T,$$

   and a matrix is unitary if and only if

   $$S^\dagger = S^{-1}.$$

3. Calculate the S-matrix for scattering from a single delta-function potential located at the origin $x = 0$.

   (All the intermediate steps have already been done for you, and can be found in the notes.)

4. Now calculate the S-matrix for scattering from a *single* delta-function potential located at the single point $x = \pm a$.

   (All the intermediate steps have already been done for you, and can be found in the notes.)

5. If S_0 is the S-matrix for an arbitrary potential $V(x)$, (of compact support), that is placed in standard position, what is the S-matrix S_a for a potential that has been shifted a distance a?

6. Calculate the S-matrix for scattering from a *pair* of delta-function potentials located at the two points $x = \pm a$.

   (All the intermediate steps have already been done for you, and can be found in the notes.)

7. Calculate the S-matrix for scattering from a two-step potential.

   (All the intermediate steps have already been done for you, and can be found in the notes.)

- End of fourth and final assignment in the undergraduate version of the Quantum module.
- If you are taking this module as part of Honours-level Applied Math, be sure to complete the additional Assignment 5.

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