School Of Mathematics, Statistics, and Operations Research
Te Kura Mātai Tatauranga, Rangahau Pūnaha

MATH 321/322/323 Applied Mathematics T1 and T2 2013

Module on Quantum Mechanics: Assignment 3

- This third assignment will deal with one-dimensional scattering phenomena described by the Schrodinger equation.
- Read chapter 5 of the notes — the chapter on one-dimensional scattering.
- Let me know of any typos or obscurities.

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1. For an arbitrary potential, calculate the determinant of the transfer matrix M.
   Be sure to simplify the result as much as possible.

2. For a pair of delta function potentials located at $x = \pm a$, complete the calculation of *all* four elements of the transfer matrix
$$M = M_{+a}\, M_{-a}.$$

3. For a single delta function potential located at the origin $x = 0$, calculate ϕ_0 the phase of the transmission amplitude t.

   How does this phase change if the delta function potential is located at $x = a$?

   Notation: Remember that for any arbitrary complex number we have $z = x + iy = re^{i\phi}$.
   The modulus is $r = \sqrt{x^2 + y^2}$ and the phase is $\phi = \tan^{-1}(y/x)$.

4. Modify the general argument regarding the location of transmission resonances for a pair of general potentials, which in the notes was given in terms of two potentials placed at $x = 0$ and $x = a$, to find where the transmission resonances should occur in the symmetric case where one considers a pair of potentials $V_{\pm a}(x)$ placed at $x = \pm a$.

5. Now use the *specific* phase ϕ_0 already calculated for the single delta-function potential, and the *general* argument regarding the location of transmission resonances for a pair of general potentials, to find where the transmission resonances should occur for a pair of delta function potentials placed at $x = \pm a$.

   Compare this application of the general argument with the explicit calculation presented in the notes.

   (You may need to track down a stray minus sign or two, and be careful about exactly where the potentials are placed.)

6. Transmission coefficients:

   (a) Show how to get from the transmission amplitude

   $$t = \frac{T_0 \exp(2i\phi_0)}{1 + (1 - T_0)\exp(2i[\phi_0 + ak])},$$

   to the transmission coefficient

   $$T = |t|^2 = t\,t^* = \frac{T_0^2}{T_0^2 + 4R_0\ \cos^2(\phi_0 + ka)}.$$

   (b) What is the maximum possible value of T in terms of T_0? When does this occur?

   (c) What is the minimum possible value of T in terms of T_0? When does this occur?

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