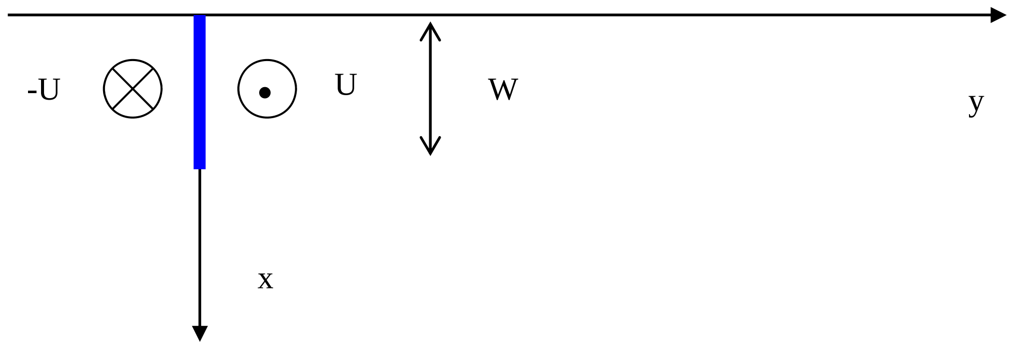
MATH/GPHS 322/323 DEs Module Assignment 2 Due Friday 24 May 2013

Q1 For the solution to the 'strike slip fault' problem



found in lectures:

$$u_z(x,y) = (U/\pi) \left[\arctan((W - x)/y) + \arctan((W + x)/y) \right] ; \quad y \neq 0$$

Take U = 1 (m) and W = 10 (km).

(i) On the same graph, plot $u_z(x,y)$, $y \geq 0$ for a range of depths x= 0, 5, 10, 20, 50 km.

(ii) On a second graph plot $u_z(x,y)$, $x \geq 0$ for a range of values of y/W = 0.2, 0.5, 1, 5, 20.

Take care with any limits as $y \to 0$.

(iii) Hence describe the general features of the displacement field $u_z(x,y)$.

Q2 For the same geometry as Q1, but with the slip on the fault described by:

$u_z(x,0^+) = (U/W)(W - x)$ on $0 \leq x \leq W$, = 0 elsewhere.

Use the general solution, equation (GS) p 6,

$$u_z(x,y) = (1/\pi) \int_{-\infty}^{\infty} y / (y^2 + [\xi-x]^2)\, f(\xi)\, d\xi \qquad \text{(GS)}$$

to find the solution by following the steps below.

(i) Show that the appropriate image slip for $x < 0$ is:

$u_z(x,0^+) = (U/W)(W + x)$ on $0 \geq x \geq -W$

(ii) Sketch the slip function $u_z(\xi,0^+)$ v. ξ for all ξ, (> 0 and < 0). This is $f(\xi)$. Over what range does the integral in (GS) need to be evaluated?

(iii) Substitute for $f(\xi)$ in (GS) and split the range of integration at $\xi = 0$. Then split each of the two integrals and solve using the method of Tutorial question 2.

(iv) Add up the integrals and compare the full solution to that for the uniform slip solution found in lectures. Briefly discuss the effects of the terms which are additional to those for the constant slip model of Q1.

(v) Show that $u_z(x,y) \to 0$ as $y \to \infty$, and that $u_z(x,y) \to f(x)$ as $y \to 0$.

(vi) On the same graph, plot $u_z(x,y)$, $y \geq 0$ at $x= 0$ for both the Q1 solution and the new solution.

(vii) Would you be able to make surface measurements of co-seismic displacements that could differentiate between the uniform slip model of lectures and this new one if you did not know W?

(viii) The fault-slip model $u_z(x,0^+) = (U/W)(W - x)$ on $0 \leq x \leq W$ is not very realistic, as it would have the maximum amount of slip on the Earth's surface, and this generally does not happen.

Describe *briefly* how the method of this question could be extended to a more realistic trapezoidal slip distribution of the fault:

$u_z(x,0^+) = U\,(x/a)$ on $0 \leq x \leq a$,

$u_z(x,0^+) = U$ on $a \leq x \leq b$,

$u_z(x,0^+) = U - U\,(x - b)/(W - b)$ on $b \leq x \leq W$.

Do *not* solve the problem! It will be helpful to sketch this new $u_z(x,0^+)$.

MATH/GPHS 322/323 DEs Module. Tutorial exercises for Friday 17 May

1. Find the limits of

$$f(y) = y \log\left(1+ (a/y)^2 \right) / \left(1+ (b/y)^2 \right)$$

as $y \to 0$ and $y \to \infty$.

2. By making a suitable change of variable, integrate:

$$I = \int_0^a y\,(a - \xi) / \left(y^2 + [\xi-x]^2 \right) d\xi$$

3. Solve the 'buried fault' problem i.e. with the same geometry as in lectures, the slip distribution is

$u_z(x,0^+) = U$ on $a \leq x \leq W$.